ROTARY POWER LAWN MOWER NOISE

A THESIS

Presented to

The Faculty of the Division of Graduate

Studies and Research

by

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In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

Georgia Institute of Technology

August, 1973

ROTARY POWER LAWN MOWER NOISE

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ACKNOWLEDGMENTS

Special thanks go to Dr. K. C. Williams, my faculty advisor, for his helpful advice throughout the project. In addition to his academic assistance he was a constant source of moral support.

For their comments, thanks go to the members of my thesis reading committee, Dr. C. R. Bragdon, Dr. David M. Sanborn, and Dr. Samuel V. Shelton. I also wish to extend thanks to the School of Mechanical Engineering for the use of their facilities and to Mr. Gene Clopton, the School's electronic technician, for his invaluable comments and assistance.

Thanks go to Sears Roebuck and Company for supplying a mower and to Mr. Howard R. Johnson of the Hush Company for supplying a muffler.

Special thanks are extended to my wife, Agnes, for her assistance in taking data, typing the draft copy, and also for her patience and moral support.

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NOMENCLATURE

Symbol	Description
A	complex pressure
B	complex pressure
B	number of blades
C	capacitance
C	compliance
E	modulus of elasticity
F	force
I	moment of inertia
M	inertance
P	acoustic pressure
R	acoustic resistance
S	area
T	transmissibility
T	temperature
V	volume
X	valve displacement
X	reactance
a	bending mode constant
c	damping
c	speed of sound
c_o	compliance
d	blade thickness

Symbol	Description
f	force
f	frequency
i	current
j	$\sqrt{-1}$
k	spring constant
k	wave number, $\frac{2\pi f}{c}$
k	Strouhl number
l	blade length
m	area ratio
m	mass
n	denotes mode of vibration
p	acoustic pressure
q	charge
r	resistance
t	time
u	volume velocity
v	velocity
v	voltage
v	blade speed, inches per second
w	angular frequency
x	displacement
z	impedance
α	acoustic absorption coefficient
α_+	transmissibility
η	blade weight

Symbol	Description
Ω_{x1}	blade enclosure natural frequency
Ω_{x2}	isolator natural frequency
ρ	density
ω	angular frequency
Ω	blade speed, reductions per second
RPM	reductions per minute
SPL	sound pressure level
PTS	permanent threshold shift
TTS	temporary threshold shift
CHABA	National Academy of Science Committee on Hearing, Bioacoustics and Biomechanics
OHSA	Occupational Health and Safety Act
DRC	damage risk criteria
Hz	Hertz, cycles per second
OPEI	Outdoor Power Equipment Institute
ANSI	American National Standards Institute
SAE	Society of Automotive Engineers
HUD	U. S. Department of Housing and Urban Development
IC	internal combustion
EPA	Environmental Protection Agency
B&K	Bruel and Kjaer

SUMMARY

The purpose of this study was to examine the noise of a rotary power lawn mower. Hearing loss and annoyance criteria were examined to determine acceptable mower noise limits. It was found that if reduced from its present level of 91 dBA at the operator to 85 dBA, mower noise would pose almost no damage risk for a typical exposure.

When the spectral content of the mower noise had been determined it was found that structural radiation was responsible for most of the noise in the range 500 to 10,000 Hz and that both the blade and exhaust were major sources below 500 Hz. The spectral content of the noise when mowing was found to be similar to that when not mowing.

The three major noise sources, exhaust, blade and structural vibrations, were studied independently. Sources of blade noise were identified by applying the theories of propellor and fan noise generation. Of the blade parameters studied, which included fan lift, sharp trailing edge and multiple blades, none reduced the A-weighted level of the blade noise.

A 1 dBA reduction was observed when the blade was statically balanced. A vibration isolator was designed and built to reduce the vibrations of the blade enclosure and resulted in a 2 dBA reduction.

Existing theories of exhaust system design were researched and used to design reactive type systems. Within the assumption of classical acoustics, no acceptable solutions were found. Applying the theory of acoustic radiation from a hole, a system was designed, built and tested which resulted in an excess attenuation over the stock muffler of 2 dBA.

With the isolator, a blade with less lift and the improved muffler, the final mower noise level was 86 dBA at the operator. Further noise reduction was limited by the noise of the blade and vibration induced radiation noise. Cost of the solutions is estimated at less than $10 and none of them are expected to impair the operation of the mower.

CHAPTER I

PROBLEM FORMULATION

Definition of the Problem

Consumer product noise is an annoying by-product of our mechanized society. Devices powered by small internal combustion engines such as generators, chain saws, model aircraft, lawn mowers, tillers, snow blowers and portable pumps are particularly annoying. Most of these engines are of the single cylinder, four-cycle, air cooled type. More than half of them are used to power the estimated seventeen million rotary power lawn mowers* in use today [1].** A solution to the mower noise problem would be a big step toward quieting all small internal combustion engine devices.

The objective of this study is to determine acceptable noise limits for the rotary power mower and then determine the quantity and quality of mower noise emitted. When the noise sources have been defined, noise abatement equipment will be designed, recognizing necessary economic constraints, that will meet or exceed the design requirements. The final stage will be the building and testing of this equipment when

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*For the remainder of this paper, "mower" will be taken to mean "rotary power lawn mower."

**Numbers in brackets refer to references listed in the Bibliography.

used on the mower.

History of Lawn Mower Noise Control Studies

To date there has been some eleven investigations of lawn mower noise. Table 1 is a tabulation of some previously reported mower noise studies and their results. The Mohr report is the only one in the group that was done for the mower industry and "represents several years of [noise] research work" [3]. The noise of a riding rotary lawn mower was studied in some detail by Faulkner [8] of Ohio State University. He arrived at the following conclusions.

1. Blade noise is important after the muffler is improved.
2. Vibrating loose parts such as fenders and seats were significant noise contributors.
3. A dynamically balanced engine results in a 4 dB improvement at a cost of $12.
4. Noise radiation from the engine casing is important.
5. Noise levels increased about 1 dBA per 200 rpm.

Conclusions in the various reports are not in agreement. The one thing that the reports do show is that there is no agreement as to what limits the noise, the engine or the blade, or a combination of the two.

Many sources of variation are possible in the data of Table 1, the more important of which include rpm, blade size, microphone position, engine load, and the environment, specifically the grass. Prout [9] has shown that the acoustic

Table 1. Lawn Mower Noise Study Results

Author	Ref	Date	Model and No.	RPM	Noise at Operator	Comments
Sperry and Sanders	2	1959				Studied blade noise
Mohr	3	1961	Lawnboy 5250	3000	82 dB SPL	Blade noise dominant
Mohr	3	1961	Lawnboy 5210	3000	85 dB SPL	Vibration important
Shearer and Stephens	4	1968			92-105 dB	Noise could be a health hazard
Hemond	5	1970			92 dBA	
Cohen	6	1970			94 dBA	Noise questionable hazard
Wyle Labs	1	1971			92 dBA	13 dB reduction at 50 ft by 1980 is possible
Pope	7	1972	Gen Leisure Prod	2780	88.5 dB	Perfect muffler will get 1 dB reduction
Faulkner	8	1971	Riding Mower		93 dBA	See text
Clark			Sears Eager 1	3000	90 dBA	
Clark			Black and Decker	3870	89.5 dBA	Electric

absorption, α, of grass can change from .3 to .78 at 1200 Hz with changes in moisture content, type of soil, and length of grass. The absorption of grass increases with frequency. In future research of this type, where the ground absorption is not known, it seems more logical to give sound pressure levels for a mower operating on a concrete surface, $\alpha = .01$ to $\alpha = .03$ [10] or some controlled environment and to specify all of the parameters mentioned above.

Recent trends in consumer product marketing are toward specifying the sound power emitted. Sound power is generally independent of the environment, however, if the acoustic characteristics of the environment are known the sound pressure level may be calculated from the sound power level. This hardly seems practical for mower customers who are interested in noise or annoyance rather than the sound power. In this respect, specifying sound pressure levels in some standard reproducible environment seems more practical.

Previous studies imply that solution of the mower noise problem requires a coordinated effort directed toward the entire mower system. It seems unlikely that there is such an effort by the lawn mower industry. Seldom does a power mower manufacturer produce his own engine, but rather he buys one from an engine manufacturer and tends to attribute the noise of the entire unit to the engine [1].

Except for liberal manufacturing society recommendations to be mentioned later, there seems to be little incentive for mower manufacturers to do anything about noise. National Civic Review [11] mentioned an instance ten years ago where a manufacturer's "quiet" mower was rejected by the public who tended to equate power with noise. That situation is undoubtedly changing as consumer opinion and the law changes.

Noise Abatement Objectives

The general noise problem is one of reducing noise to reduce annoyance, hearing damage, or both. Noise from traffic or office machinery is usually reduced to eliminate annoyance, while that from a punch press in a worker's environment is reduced to prevent any worker's hearing loss. Aircraft and lawn mowers are examples where both annoyance and hearing damage may be important.

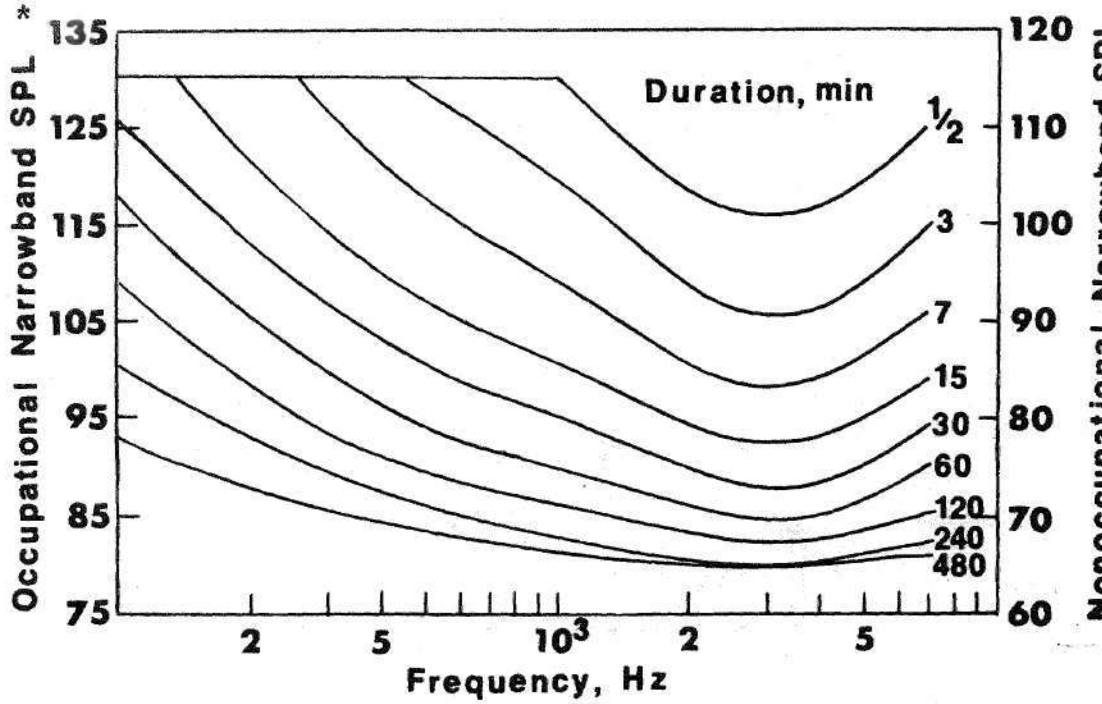
Hearing Damage

Hearing damage can be related to noise exposure through a damage-risk criteria which specifies the maximum noise levels and exposure time to which a group may be exposed such that a certain percentage of the group suffers no hearing handicap. A person's hearing is considered impaired when there has been a noise induced permanent threshold shift, PTS, of 15 dB or more at 500, 1000, and 2000 Hz [12,13]. One of the most widely accepted damage-risk criteria is that prepared by Working Group 46 of the National Academy of Science Committee on Hearing Bioacoustics and Biomechanics, the CHABA method [13].

One of the postulates in the CHABA criteria is that a worker's temporary threshold shift, TTS, is not only a consistent measure of a single days exposure to noise, but a measure of the hazard associated with years of such exposure. Hence, it is their assumption, that daily TTS greater than 15 dB at 500, 1000, and 2000 Hz will produce, after many years, a 15 dB PTS at those frequencies and a hearing handicap.

By comparing the TTS that young workers suffer after exposure to their working environment to the PTS suffered by older workers in similar environments, the CHABA group was able to determine what noise levels and exposures would produce acceptable TTS in 90 percent of a group of workers. Their results are presented as a Damage Risk Criteria, DRC contours, a variation of which is shown in Figure 1. Its use requires that the frequency spectrum of the noise be established. Where this is not possible, the A weighted sound level may be used to specify permissible noise exposures. These criteria were incorporated into the Walsh-Healy Act and the Occupational Health and Safety Act, OHSA. Permissible exposures according to OSHA are shown in the occupational column of Table 2.

The damage-risk criteria used in OHSA only protects about 90 percent of an affected group [11] and offers no protection to frequencies above 2000 Hz. It also assumes



*All sound pressure levels in dB re 2 x 10^{-4} μ bars.

Figure 1. Damage-Risk Contours for One Exposure per Day to Octave Bands Present in Broadband Noise [13]

Table 2. Permissible Noise Exposures

dBA	Limiting Daily Exposure Times Non-Occupational [6]	Occupational [12]
115	less than 2 min	less than 15 min
110	4 min	30 min
105	8 min	1 hour
100	15 min	2 hours
95	30 min	4 hours
90	1 hour	8 hours
85	2 hours	
80	4 hours	
75	8 hours	
70	16-24 hours	

that the worker has a rest period in which to recover. Cohen, Anticaglia, and Jones [6] consider the criteria acceptable for industry where the worker is paid for assuming job risks and can be compensated should occupational related hearing loss occur, but unacceptable for off the job situations. They have proposed the exposure limits for nonoccupational related noise shown in column 2 of Table 2. If those exposure limits were followed, no hearing loss would result from nonoccupational noise insults for nearly 100 percent of the exposed population for the frequency range 500 to 6000 Hz [6].

Recognizing that mower noise may be harmful, a study was undertaken at Northern Illinois University [4] to determine the TTS in fifteen subjects who were subjected to mower noise for 45 minutes. The mower noise that they were exposed to was about 97 dBC. TTS ranged from 0 to 35 dB with the averages as shown in Figure 2. Another study by Cohen et al. [6], noted nearly the same TTS for a similar exposure. Based upon these studies it may be concluded that daily exposure to mower noise does not pose a large damage-risk to hearing. Those who suffered a 35 dB TTS or who were recovering from a previous noise exposure could possibly be affected by the mower noise.

The National Industrial Pollution Control Council [14] has proposed the following graduated reduction of lawn mower noise when measured at the operator in its normal mode of

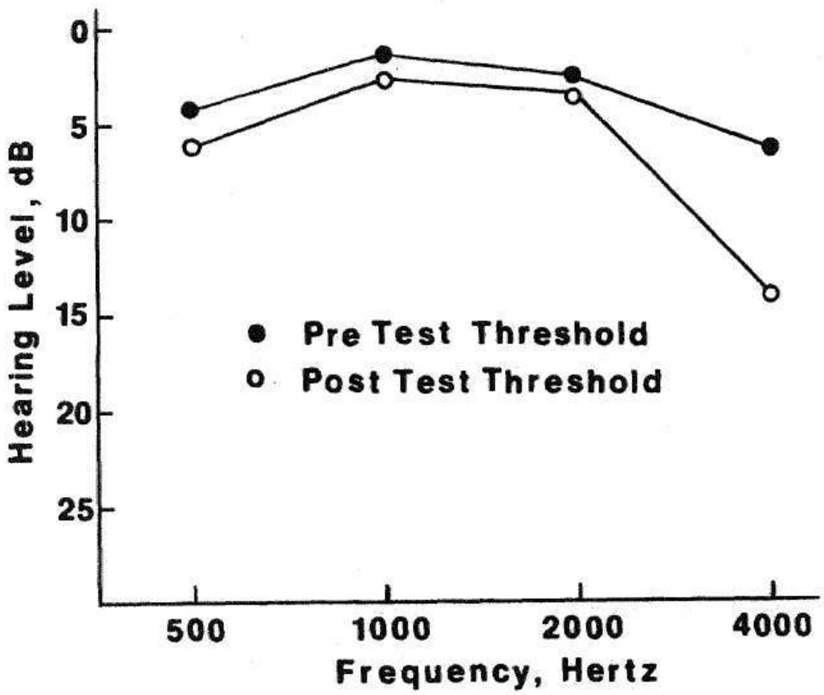


Figure 2. Temporary Threshold Shift of Mower Operations [5]

operation: 1970, 92 dBA; 1978, 85 dBA; 1983, 82 dBA. The Outdoor Power Equipment Institute, OPEI, in recommendations published by the American National Standards Institute [15], ANSI, has established 92 dBA as the maximum permissible noise level at the operator for certification of a mower by that group. After inspection of Table 2, a reduction of 2 dBA to 90 dBA seems reasonable from the point of view of protecting both the once-a-week home operator and the professional.

Annoyance

In addition to its potential harmful effects, noise can be annoying. It can often interfere with sleep, communication, work, and mental health. Unlike the noise limits set to prevent hearing loss, it is difficult to determine specific acceptable limits in sound pressure level below which there will be no annoyance. Annoyance is a complex function of ambient noise level, quality of the noise, what produces the noise, when the annoyance occurs, how often it occurs, what it interrupts, and the person's attitude toward the noise source. Tones in a noise such as turbine whine in aircraft, propellor noise, or the hum of a lawn mower blade make some noises particularly annoying [16]. About ten different noise rating systems have been developed for use in various applications, but the A weighted sound level is the easiest to use and seems to correlate well with other noise ratings [12].

It could be argued that lawn mower noise is not a major source of community annoyance. Only two out of 1745 complaints by Chicago residents in a recent nineteen-month period concerned lawn mower noise [17]. This may be partially due to the reluctance of people to complain about their neighbor when they themselves probably have a lawn mower.

Public awareness and resentment of environmental noise is evidenced by an increasing number of city ordinances and state and federal laws and guidelines. A City of Chicago ordinance specifies the following noise level limits on small noncommercial power equipment when measured fifty feet from the product in accordance with SAE Standard J952b [18]. Manufactured after January: 1972, 74 dBA; 1975, 70 dBA; 1978, 65 dBA. A City of Minneapolis ordinance [1] restricts the hours during which a device may be operated if its noise measured at the property line exceeds the ambient by more than 6 dB.

Based on social surveys, the United States Department of Housing and Urban Development, HUD, has adopted a set of guideline criteria for noise exposure at residential sites [12]. These criteria are expressed in terms of outdoor and indoor A weighted levels not be be exceeded for so many minutes per twenty-four hour period. When the Chicago-SAE 1978 requirement of 65 dBA at 50 feet is examined in the light of the HUD criteria it would be judged as clearly acceptable.

Summary

The noise levels required at the operator for no hearing loss are not compatible with the annoyance noise requirements measured at 50 feet. Although it will be shown to be true later, assume for the moment that the mower noise decreases 6 dBA for each doubling of distance from the source and that it is nondirectional. For 65 dBA at 50 feet there would be 71 dBA at 25 feet, 76 dBA at 12-1/2 feet and about 85 dBA at the operator. Satisfying this one constraint of 85 dBA at the operator would satisfy both the annoyance and hearing loss criteria provided the 6 dBA per doubling of distance assumption is true and the power noise is nondirectional. This is admittedly conservative and proclaimed by the mower industry as difficult to reach [17]. Attainment of this goal would result in practically no hearing loss or annoyance from a rotary power mower.

Other Design Considerations

No attempt will be made in this report to optimize schemes of home beautification that might eventually lead to the elimination of the present lawn mower. Although artificial turf and grass that does not need mowing may be available, most people presently have grass that grows and must be mowed.

The electric lawn mower is probably cheaper to buy, operate, and maintain but its blade may make more noise than that of IC engine powered machines, resulting in comparable noise levels for both types (see Chapter IV). Black and Decker's cordless electric reel type mower might be an excellent solution to the noise problem. Other grass cutting techniques that have been tried that may make less noise are sicle bars, water jets, hot wires and laser beams [19].

Rotary lawn mowers are currently the most popular lawn cutting device. They can cut longer grass to a more uniform length than reel type mowers and grass clippings can be easily collected. There are certain design constraints however that make the mower inherently noisy. The blade must turn fast enough to store energy to cut dense clumps of grass and keep horsepower as high as possible. It must rotate fast enough to cut the grass instead of knocking it over and to discharge clippings with sufficient velocity to spread them evenly. According to the Toro Corporation, 3000 RPM [19] is the minimum acceptable blade speed for normal length blades. OPEI recommends a maximum blade tip speed of 19,000 feet per minute.

The blade must be covered with a shroud to direct the clippings, hold the wheels and engine, and for obvious safety reasons. The blade must have fan type ends that will provide a suction to pick up matted down grass for cutting and to discharge clippings into a collector bag if necessary without plugging the exhaust shut. Toro [19] considers air turbulence noise important in retarding grass buildup on the inside of the blade enclosure. Safety requirements for the entire mower system are clearly outlined in ANSI B71.1-1972.

The primary constraints on any modifications to the engine concern cooling and the exhaust. Any enclosure must provide for cooling air to enter and leave. The exhaust system should introduce a maximum back pressure to the exhaust port of 10 inches of water [20]. The muffler's effects on scavenging and the engine's respiration is also important. The muffler should be small enough not to interfere with the mower's operation and be a safety hazard.

It should not render the system aesthetically displeasing to a potential customer.

The most important constraint is cost. Except for isolated cases, people, generally uneducated concerning the hazards of intense noise exposure, are probably more concerned with mower cost than with noise. A potential mower customer aware of the hazards of noise is the United States Government. The Noise Control Act of 1972 allows the government to pay up to 25 percent more for low-noise-emission products [53]. The mower industry is fairly competitive, having about 50 manufacturers in the United States. At the present, noise levels for lawn mowers are only recommendations. Manufacturers can not be expected to raise prices if necessary for a quieter design, unless there is sufficient public demand for the product that will result in economic rewards for the company.

Although an individual may have an exotic quiet mower, he will still have to endure the noise of his neighbors. The neighbor, if not bothered by the noise himself, cannot be expected to bear the cost of a device that will benefit someone else. Without regulation, the only sure way to rid annoying noise from a product such as a mower is to make a quiet model cheaper.

How much are consumers willing to pay for noise reduction? It would be convenient if this information were available, but unfortunately it is not [21].* Loudness is a

*This information is expected soon in an EPA report.

complex function of frequency and intensity as well as being dependent on consumer attitude. An attempt to arrive at a crude approximation of the consumer's willingness to pay for noise control is shown in Appendix A. From this study a solution would be judged acceptable if there was less than .3 percent increase in cost per 1 dBA reduction.

Summary of Constraints and Objectives

1. Noise levels should be 85 dBA or less at the operator.
2. All solutions should be as inexpensive as possible.
3. Lawn mower must meet OPEI safety recommendations.
4. Muffler should be as small as possible and not have more than 10 inches of back pressure.
5. Final system should be able to mow grass effectively.

Method of Attack

Before any solution to the noise problem can be sought, the sources of noise must be clearly defined. It is for this reason that preliminary results must be obtained that will identify noise sources and the relative magnitude of the noise from each source. After this has been done, each source will be considered individually. The final test for each noise solution will be its contribution to the noise of the mower system.

CHAPTER II

INSTRUMENTATION, EQUIPMENT AND PROCEDURE

Equipment

Most experiments performed during the course of this study were done on a 19-inch Sears "Eager 1," model number 131 91383 rotary lawn mower which had previously been used as a demonstrator. It was equipped with a ten cubic inch, 3-1/2 horsepower, four-cycle, air cooled Techumseh brand internal combustion engine. The motor from a Black and Decker model 8000 electric rotary lawn mower was used to accurately determine blade noise.

Various blades and mufflers were studied in the project and will be discussed in their respective chapters. It should be noted that the different blades had different fan loads resulting in changes in engine speed for a constant governor setting.

Instrumentation

Where only the sound pressure level or the A weighted level was needed, data was taken with a General Radio Type 1551-B Sound Level Meter. Where octave band data was needed it was obtained with a Bruel and Kjaer Type 2204 Impulse Precision Lound Level Meter with a Type 1613 Octave Filter Set.

Frequency analysis data was taken with a Bruel and Kjaer Type 2107 Frequency Analyzer, having a 6 percent bandwidth filter. To reduce the error in the data due to fluctuations in the indicated SPL on the 2107, a Bruel and Kjaer Type 2417 Random Noise Voltmeter was used. This allowed the use of a 10 second meter averaging time which nearly eliminated fluctuations in the indicated SPL. Data was recorded manually.

A Bruel and Kjaer Type 4134 one-half inch condensor microphone used with a Type 2615 cathode follower provided the acoustic signal. The microphone was mounted on a tripod 54 inches above the ground and 3 feet behind the mower's rear wheels. A 25 foot microphone cable allowed for the placement of the equipment well away from the mower. Although well grounded, the microphone cable had to be supported above the ground to eliminate electrical interference. The microphone was calibrated with a Bruel and Kjaer Type 4220 Pistonphone Calibrator.

Vibration data was taken with a Bruel and Kjaer Type 4336 piezoelectric accelerometer whose output was observed on the wave analyzer. In experiments on the muffler, data was taken with a Bruel and Kjaer Type 4136 one-fourth inch condensor microphone and a Bruel and Kjaer Type UA 0040 microphone probe kit. The signal to calibrate the probe was generated in a Bruel and Kjaer Type 1042 Random Generator.

At one point in the project, noise was recorded on a Model 1520 Wollensack tape recorder for analysis in the wave analyzer. This recorder has less than +2 dB error for response between 40 and 15,000 Hz. Calculations in the muffler design were performed on a Univac 1108 Digital Computer and a Model 700 Wang Calculator. A Tektronix Type 502A Oscilloscope was also employed for observing the shape of the exhaust pulse.

Facilities

Originally it was hoped that a semireverberant room could be constructed where sound power level and insertion loss information could be obtained. The space available allowed for a room 9 x 11 x 16 feet, much too small, resulting in about 15 resonances between 100 Hz and 300 Hz, frequencies of considerable interest in this study. The room was reasonably diffuse, however, ± 2 dB when excited with mower noise, and had some utility in determining insertion loss.

Most of the frequency analysis data was taken with the mower operating in a large asphalt parking lot since there was no anechoic chamber available in which meaningful data could be obtained in the low frequency range. A schematic of the experimental setup is shown in Figure 3.

Procedure

Measuring and Controlling Engine Speed

The engine speed of the IC engine powered mower was controlled with the stock mechanical governor and was

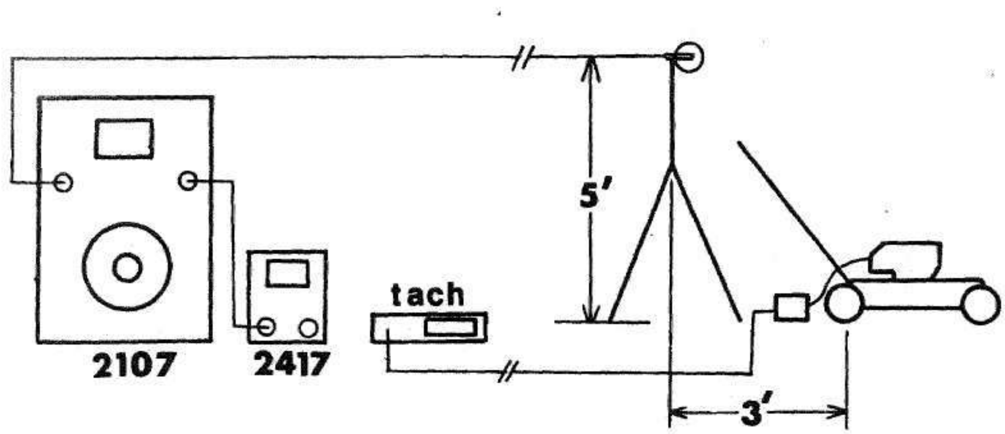


Figure 3. Schematic of Experimental Setup

monitored with a Hewlett-Packard frequency counter. The signal for the frequency counter came from a coil of wire that was placed around the spark plug wire. The signal from the coil was amplified in a battery powered, two stage, transistor amplifier before it went to the counter where the average period of revolution was monitored since this was more accurate than monitoring frequency. Peaks in the noise spectrum provided an additional check of rpm.

Engine speed was factory set near 3000 rpm, with no provision to easily change it. The stock mechanical governor held the rpm fairly constant under a constant load, but when the load changed, the governor often corrected to some new rpm. For this reason, a governor adjustment was made which allowed the mean engine speed to be adjusted whenever the load changed.

A method of setting the mean value of rpm that was unsuccessful should be mentioned. A jet was installed in the carburetor to vary the air-fuel ratio. It was hoped that by changing the amount of fuel at a constant governor setting, a controllable change in rpm would result. In this respect it worked well, but examination of the noise spectrum showed significant changes in the amplitude of the fundamental exhaust pressure peaks. It was then established, as should have been expected, that large changes in air-fuel ratio at nearly constant rpm was changing the manifold vacuum and other pressures throughout the engine.

With the engine speed controlled by the stock mechanical governor and under constant load near 3000 rpm, variations of as much as 300 rpm were observed. The standard deviation from the mean rpm, however, was 84 rpm or about 3 percent. The mean value of rpm changed about one-half percent in two minutes (see Appendix B).

Determining Blade Noise

To determine blade noise an electric motor was mounted on the mower being studied. Care was taken to insure that the position of the blade in the shroud was not changed when the electric motor was used. Motor speed was regulated with a variable transformer. Since the electric motor speed was very steady, it was not constantly monitored during testing but was set initially and checked after each test with a strobe tach or the frequency analyzer.

An unsuccessful attempt at measuring the blade noise might be mentioned. Since no electric motor was initially available, an attempt was made to run the mower without the blade. It was immediately determined that lawn mower engines will not run without the blade since the blade is used as a flywheel. The mass moment of inertia of the blade was experimentally determined and a disk of equivalent moment of inertia was made and mounted on the engine. The engine ran but without the blade fan load there was little engine load resulting in large fluctuations in rpm. Using this method did not allow repeatability of the frequency analysis data.

Determining Engine Noise

Exhaust noise was separated from engine noise in a large, 50 x 75 x 25 foot shop area. The mower was placed near a door, through which the exhaust and its noise was piped. By comparing data with the exhaust outside to that with the exhaust not piped out, an estimation of engine noise was obtained. Exhaust noise was not measured after piping it outside because the acoustic effects of the pipe were not known.

Combining Noise Levels

There are two methods by which noise levels may be combined. The first method, that of adding sound power, is applicable when the sounds being combined are nearly random noise and are not in phase. When combining the noise of two identical machines as would be properly done by this method, the sound power doubles and the sound pressure level increases

3 dB. This method is presented in chart form in reference 22.

If the levels added are of noise that consists of tones or if the sounds are in phase then the sound pressure must be added. To add sound pressure, their phase relation must be known. Using this method, the sum of two equal magnitude pure tones with a phase angle of 0 would result in an increase of 6 dB. If the tones were out of phase by 180 degrees, then their total would be zero. Note that the position of the sources and their type of radiation, plane wave, cylindrical, etc., will affect the resultant sound pressures. For further discussion of this subject see reference 12.

Care must be exercised when subtracting the individual peaks of the spectrum shown in the next chapters. Since little can be said about the phase relations between the various mower noise sources no attempt will be made to add or subtract the sound pressure levels of the spectrum presented in this paper.

CHAPTER III

PRELIMINARY RESULTS

Effect of Changing Mower Variables

RPM

Figure 4 shows the change of sound pressure level, SPL, at the operator with rpm. Since SPL decreases with rpm, and since it has been established as the minimum acceptable blade speed by Toro, all other studies in this report are performed at 3000 rpm.

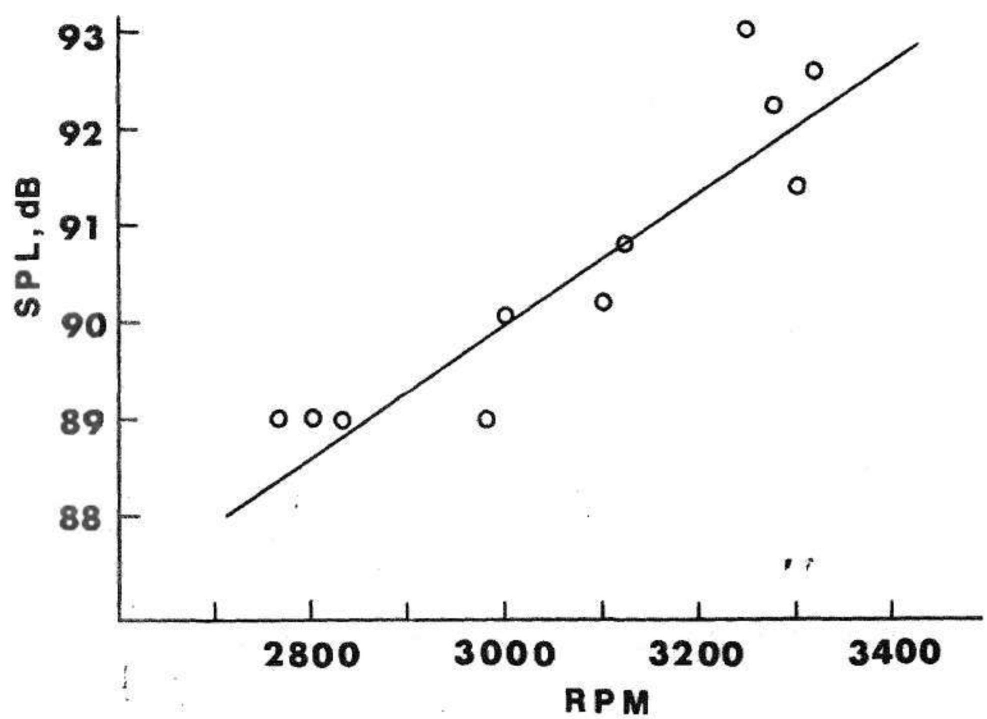


Figure 4. Systen Sound Pressure Level vs RPM

Engine Load

As the load is increased, the SPL should increase as a result of increased cylinder pressures. An engine was mounted in the semireverberant room such that a controlled load could be applied. At constant rpm the SPL increased 4 dB as the torque was increased from 0 to 3 foot pounds. All further tests in this study were done with only the fan load of the blade acting on the engine.

Mower Height

The mower was operated on asphalt and the height, the distance from the blade to the ground, was varied from 7/8 to 3-1/4 inches. The relative position of the blade in the shroud remained the same. A maximum variation in SPL and A-weighted level of 1 dB was observed as the height changed, with the maximum noise occurring between 2 and 2-1/2 inches. All further tests in this study were conducted at a mower height of 2 inches. Guenther of Ohio State University is currently studying the effects of the position of the blade in the shroud, but his results are inconclusive at this time. Every attempt will be made in this study not to vary the blade position.

Surface Changes

Table 3 shows how the sound pressure level at the operator is affected by changes in the surface on which the mower operates. Although these changes are not large, there is a tendency for the noise to be less on surfaces with

Table 3. Changes in Mower Noise with Changes in Surface

Type of Surface	Mower Noise SPL	Mower Noise dBA	Ambient SPL	Ambient dBA
Cement	92	89½	68	50
Dirt	92½	90	72	54
Asphalt	92½	90	72	54
Grass	91	88	72	54

greater acoustic absorption. To eliminate this source of error, all further experiments will be performed with the mower operating on asphalt.

What are the effects of ground absorption on the noise levels measured 50 feet from the mower? Although the absorption of blade noise by the grass and ground under the mower may be significant, Beranek [12] shows that the attenuation of noise as it passes over grass or the ground is insignificant for sound with frequencies below 10,000 Hz for distances less than 50 feet.

Directivity

The directivity pattern of the mower is shown in Figure 5. It can be seen that the mower is nondirectional and that it follows the 6 dB per doubling of distance from the source. Except for very close to the mower, closer than the operator's position, mower noise is not a function of

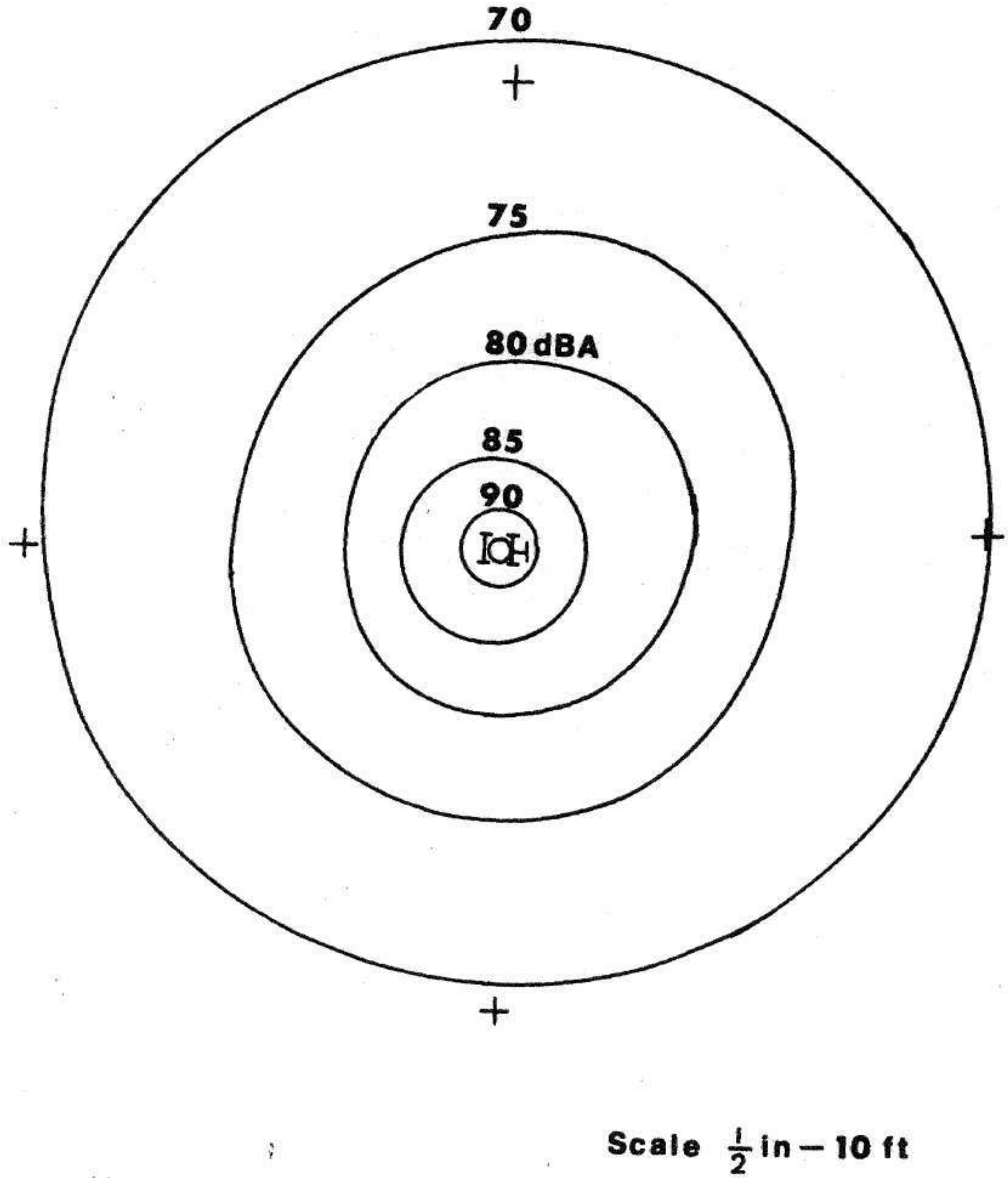


Figure 5. Mower Noise Directivity

the height above the ground.

Sources of Noise

Vibration

Figure 6 shows the frequency spectrum of the noise at the operator's position for the stock internal combustion engine powered mower. The overall ground pressure level, the total level of the sound between 2 and 20,000 Hz, is indicated on the axis of Figure 6. Since the mower seemed to be vibrating excessively, the blade was checked and found to be out of balance due to uneven sharpening. After the blade was statically balanced the spectrum remained essentially the same. The peaks at 100, 125 and 150 Hz were reduced about 2 dB each and the A-weighted level dropped 1 dB to 90 dBA.

Exhaust

The most difficult noise to separate was the exhaust noise. Figure 7 shows the effect of piping the exhaust outside away from the system. The result is a 1 dB drop in SPL. This agrees with the conclusions of a similar study by Pope [7]. Insertion loss measurements in the semireverberant room show that with a special muffler, to be discussed later, the SPL dropped 5 dB. The only conclusion possible here is that a better muffler will certainly help, although how much is not known.

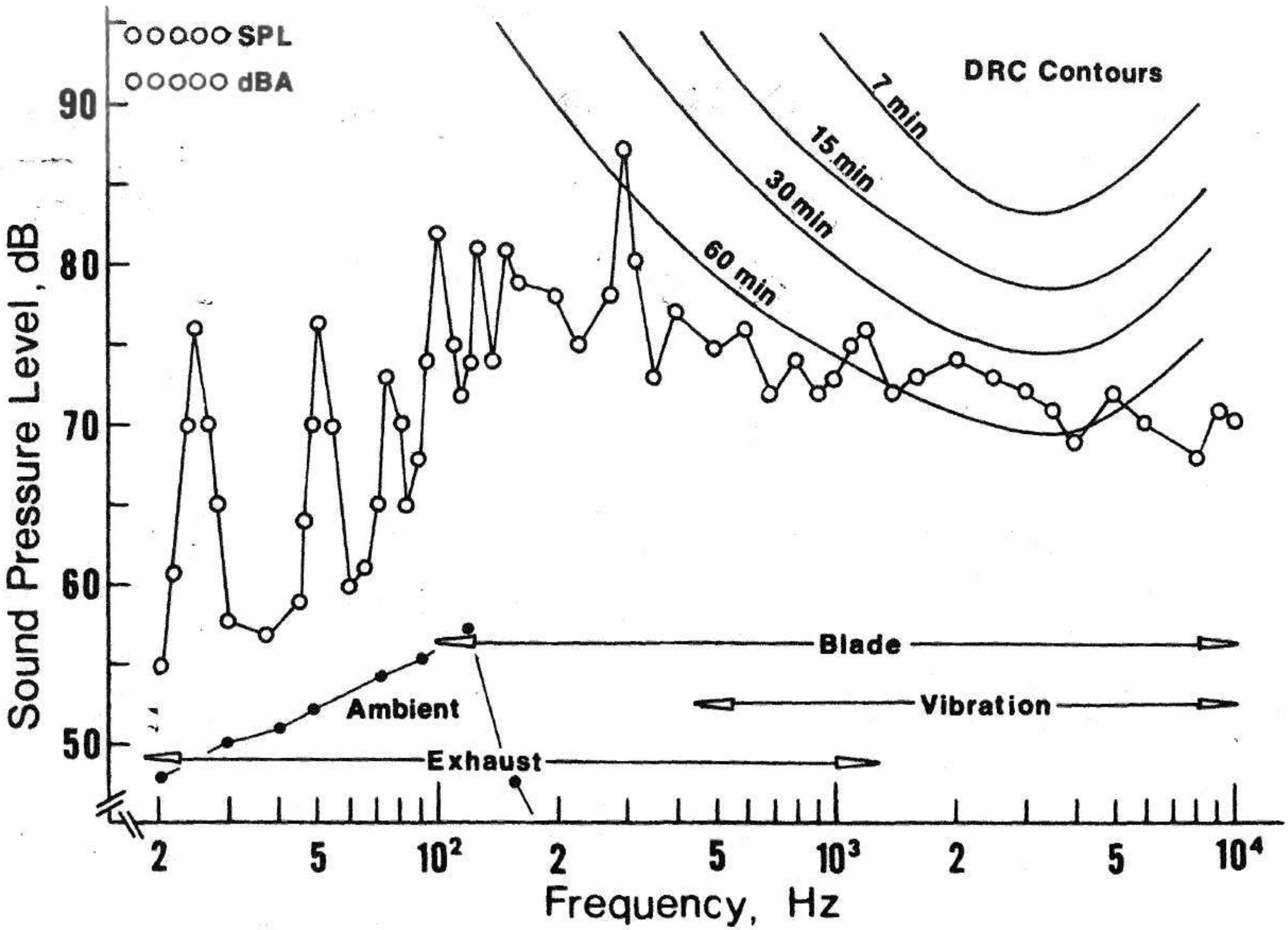


Figure 6. Mower Noise Spectrum

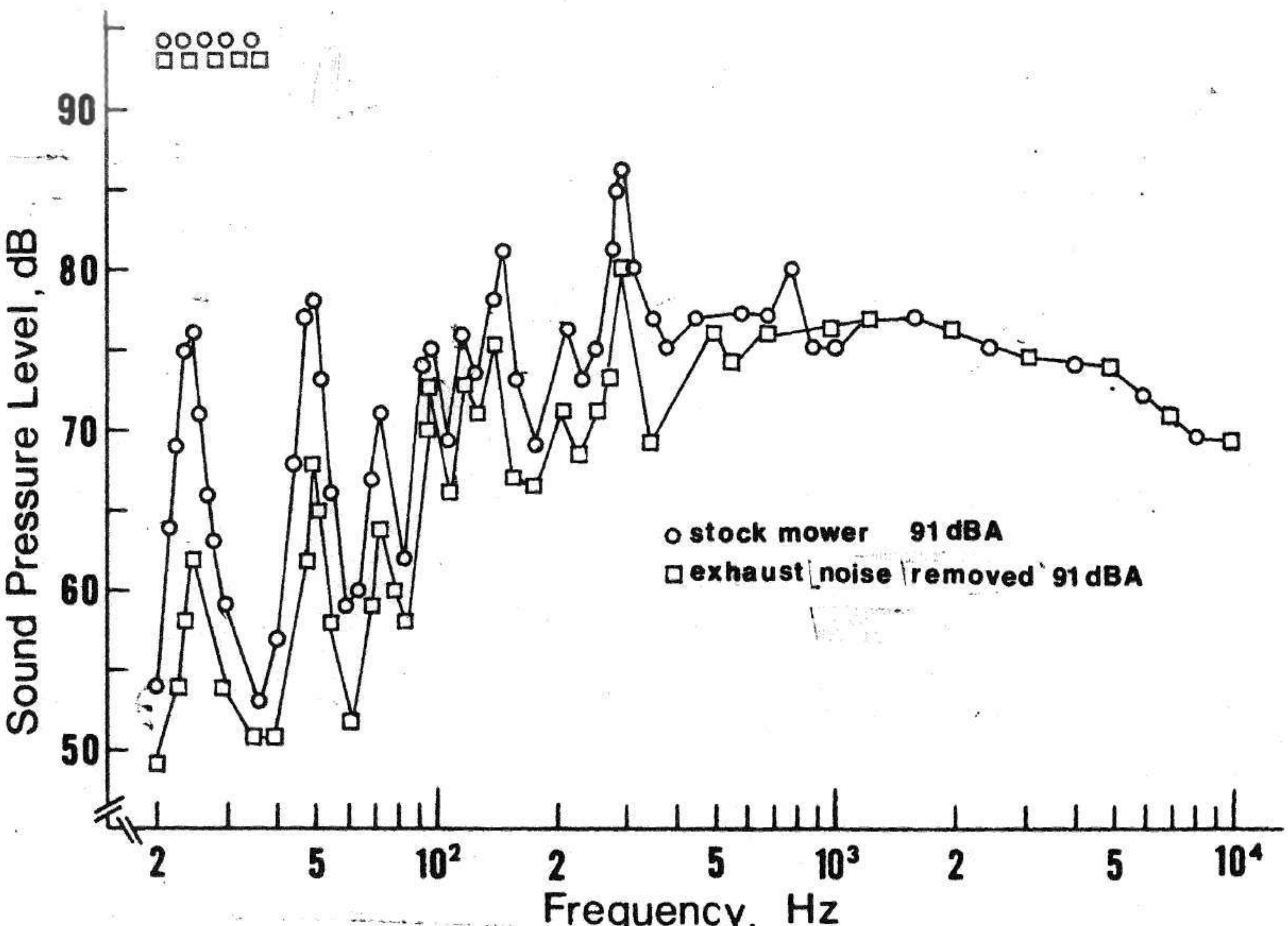


Figure 7. Typical Mower with Exhaust Noise Removed

Blade

This is the easiest noise to determine. Figure 8 shows the significance of the contribution of the blade noise to the complete mower noise spectrum. Blade noise is a significant component since the noise of the complete mower is only 4 dB greater.

The Noise of Cutting Grass

A study of mower noise would not be complete without investigating the noise of cutting grass. Recalling what has been said about the variation of acoustic absorption of grass, and the change in sound pressure level with load and rpm, the problem of finding the noise of cutting grass can be appreciated.

This noise source was investigated by mowing grass of a constant height (grass that had already been mowed) with the electric powered mower. Sound pressure level as well as A-weighted levels differed little when mowing or just running on grass. Since the ear can hear distinct changes in the noise when mowing, further data was taken to try to explain the change in noise quality.

Figure 9 shows octave band data taken while mowing. Also plotted are octave band data for the mower operating in dense grass and on cement. This shows the absorption of blade noise by the grass at high frequencies as expected since the absorption of grass increases with frequency [9].

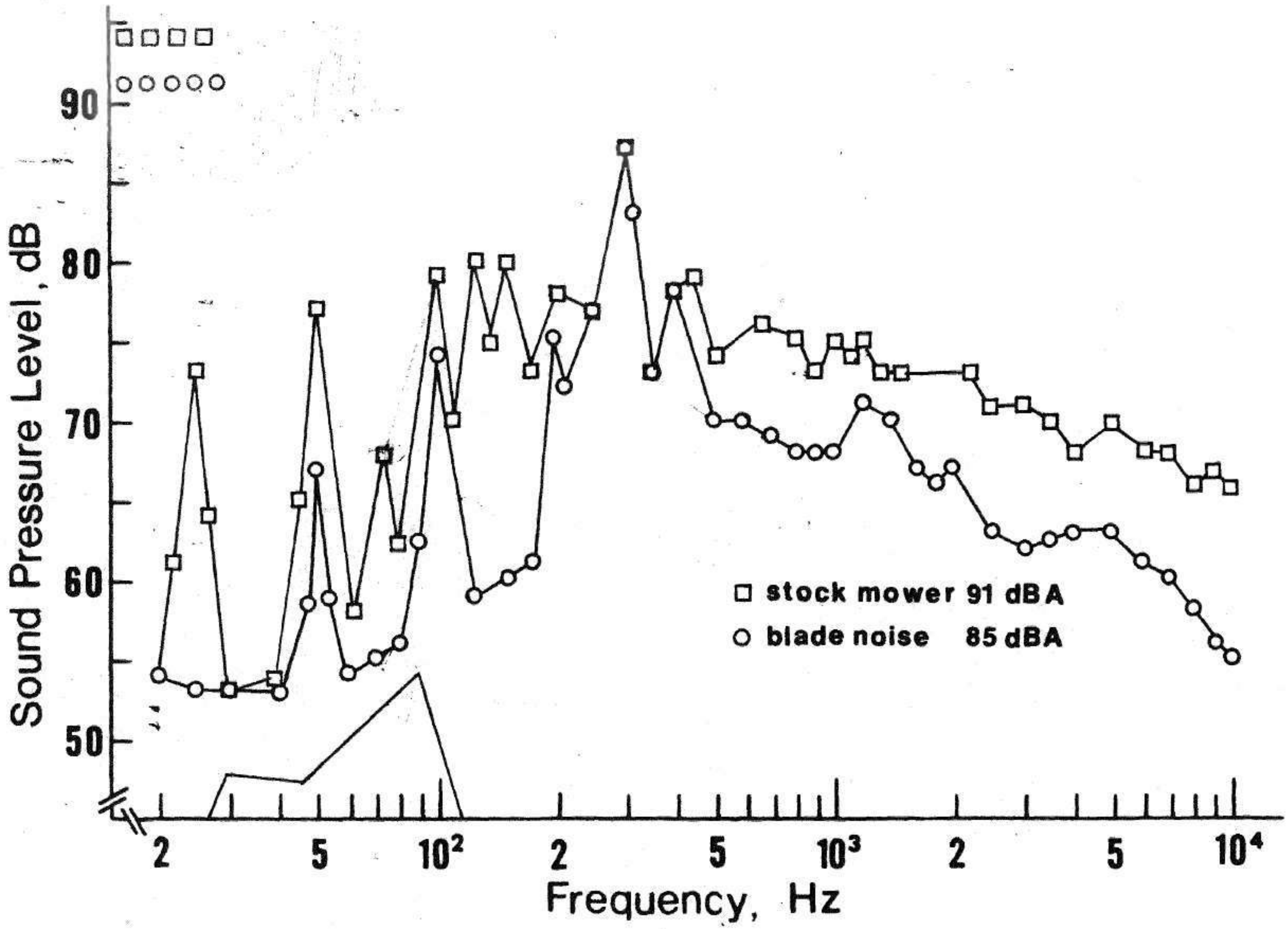


Figure 8. Stock Mower with Blade Noise

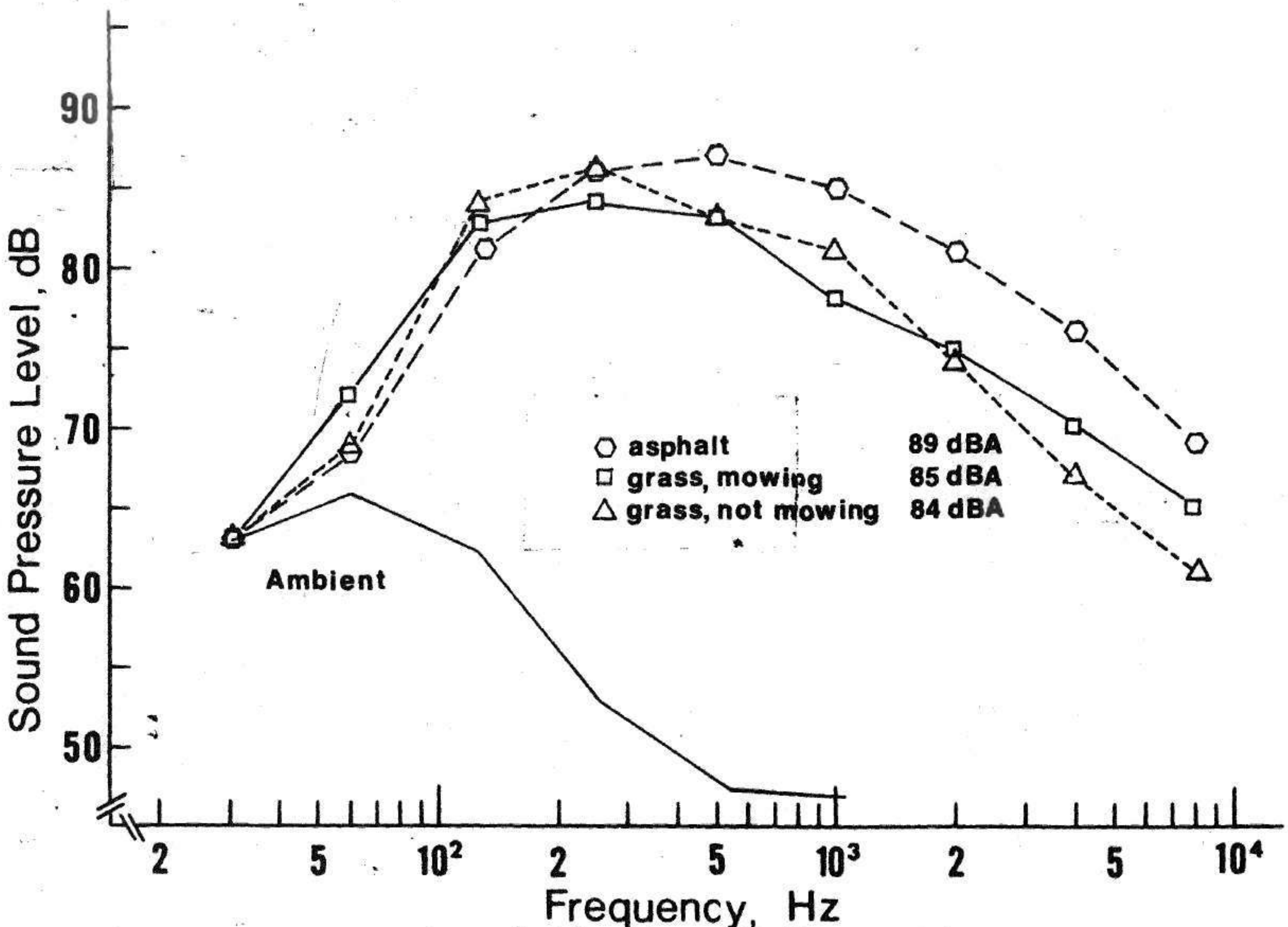


Figure 9. Noise of Mowing Grass

The data shows little change in any of the octave bands when mowing.

To further investigate the noise change when mowing, the noise in the cutting and noncutting modes was recorded in a loop on the tape recorder, played back through the wave analyzer and the frequency spectra of Figure 10 were obtained. Note that since the tape recorder was not calibrated the spectra only shows relative magnitudes, not absolute sound pressure levels. Although there is error present as a result of changes in rpm and the fact that the recorder was not calibrated nor of laboratory quality, the results agree quite well with the octave band data in that the noise between 2000 and 10000 Hz has increased when mowing and the low frequency noise has also increased.

The following observations will be noted from the spectra. First, the curves taken while mowing are smoother, the low frequency tones having been modulated somewhat making them less annoying. Second, one of the low frequency tones, that expected at 500 Hz, completely disappeared when mowing.

Although the spectra help to explain why the noise when mowing sounds different, no attempt to further explain these changes will be made in this report. Because of its transient nature, further study of this noise source should be done with a real time analyzer. The important conclusion to be drawn from the examination of grass cutting noise is that there are no major changes in the noise levels or

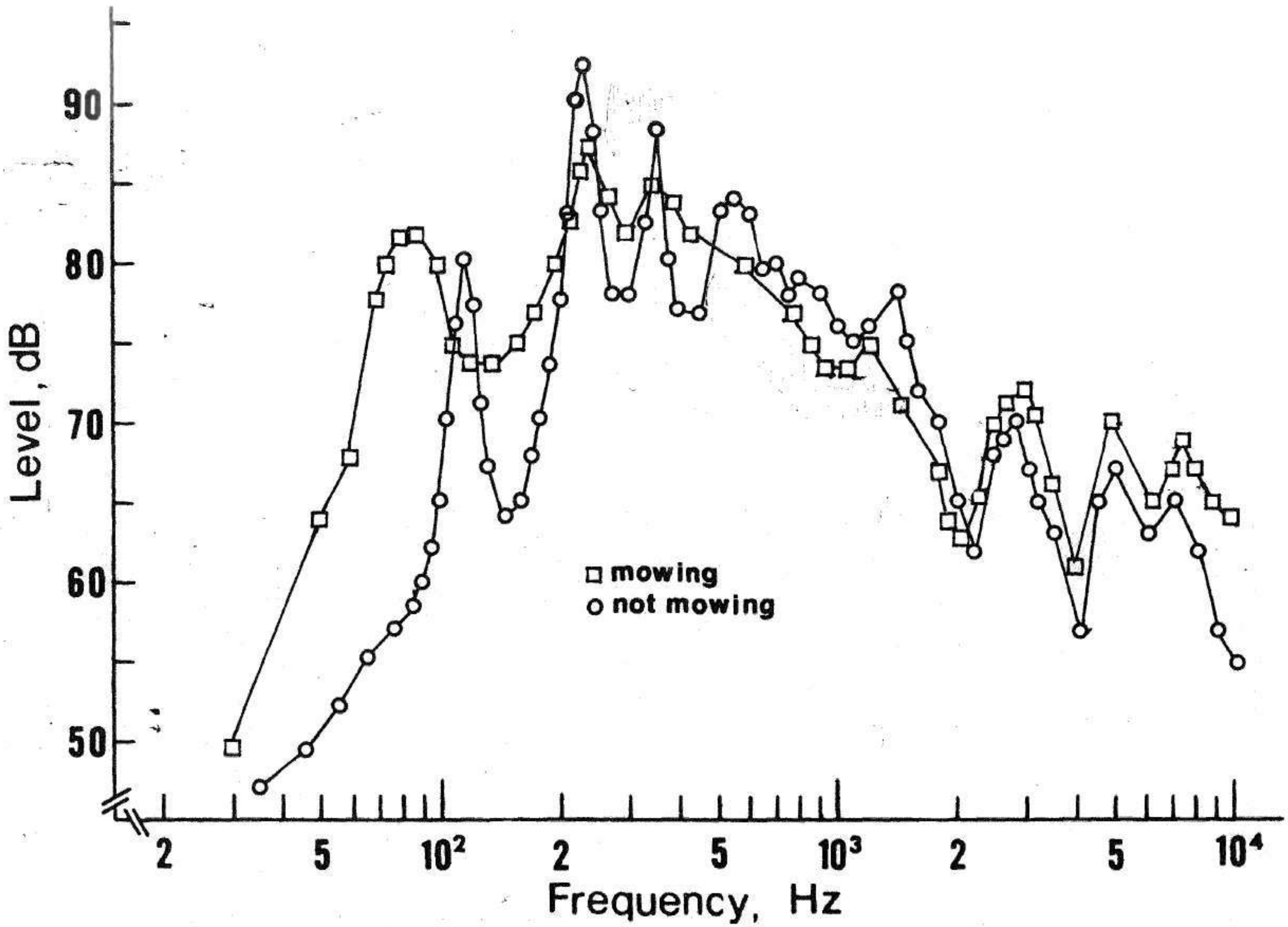


Figure 10. Spectrum of Mowing Noise

spectrum when mowing. Data and conclusions obtained for a mower operating on asphalt will still be useful in the solution of lawn mower noise.

Error and Repeatability of Data

There are five important sources of error in the frequency analysis data, variations in rpm during a run, variations in rpm between runs, variations due to calibration of the equipment, frequency variation of the signal caused by fluctuating rpm results in error in the analysis due to filter bandwidth, and error due to placement of the microphone.

If a least square straight line is fit through the data points in Figure 4, it has a slope of .00699 dB per rpm. The standard deviation of the rpm is about ±84 rpm, hence the maximum expected deviation in SPL due to variations in rpm will be about ±.6 dB at 3000 rpm.

Error is also introduced as the mean value of rpm changes between runs. Although an effort was made to prevent this, a variation of about 100 rpm may have occured. From Figure 4, this would result in an error of about ±.75 dB.

Using the pistonphone, the microphone can be calibrated within less than .1 dB. Error introduced because of the equipment will be considered negligible.

The fact that the bandwidth of the wave analyzer is 6 percent and the rpm is varying about 2 percent results in a constant reduction in levels in the data. An example will help to explain this and will also show the advantage of the constant percentage bandwidth type filter when analyzing fluctuating frequencies. Consider a constant bandwidth type filter 5 Hz wide. When analyzing a signal at 300 Hz, it finds the level of all of the frequencies in the range 297-1/2 to 302-1/2 Hz. The 6 percent bandwidth filter examines those frequencies in the range 291 to 309 Hz. Assume the noise source produces a tone at 300 Hz but due to the noise source's 2 per cent fluctuation in rpm, the tone is modulated between 294 and 306 Hz. It should now be clear that in this case the constant percentage bandwidth will yield a more precise result. Scott [23] shows the error of a 6 percent bandwidth filter when measuring the amplitudes of a tone modulated ± 1 percent to be 3 dB. This error is not a function of frequency and is constant for a 1 percent modulation. For the constant bandwidth filter this error increases rapidly with frequency. The error from this source should not vary between runs unless the amount of rpm fluctuation changes.

To reduce the error from the placement of the microphone, it was always mounted on a tripod a measured distance from the mower. Changes in SPL within 6 inches of the microphone position were not measurable, although at distances greater than 6 inches from it changes in SPL could be observed. The maximum error introduced here is less than 0.5 dB.

The above five sources of error will result in a maximum error in the SPL presented in the frequency spectra of about ±4.5 dB and a variation between runs of about 1.5 dB. Measurement of overall noise levels does not include the effects of frequency modulation and is more repeatable than the levels in the spectra, having a tolerance of about ±1 dB. The repeatability of a typical data set is shown in Appendix C. SAE recommendation J952b allows a tolerance of 2 dB in the measurement of overall levels.

The above discussion did not consider variation in the data when tests were performed on the electric mower. Results on that mower were much more repeatable, having a tolerance of about 1 dB for spectral data and 0.5 dB for overall levels. Note that there may be error due to frequency modulation when comparing this data with that of the IC engine powered mower.

Preliminary Conclusions

From the preliminary results, the following conclusions are evident. Lawn mower noise is a function of load, rpm, and air fuel ratio although this variation will not be studied here. It is not directional, does not change with mower height and is dependent on the surface upon which the mower operates. Noise levels on asphalt or cement will be about 3 dB greater than those levels measured on grass. Overall noise levels do not change more than 1 dB and the spectral content of the noise does not change significantly when mowing.

To reduce the effects of those things that affect mower noise and introduce experimental error, further tests will be made with the engine loaded only by the blade fan load, at 3000 rpm, and on asphalt pavement. Measurements taken under these conditions are repeatable within ± 1.5 dB.

Mower noise can be attributed to three important sources, exhaust, blade, and radiation resulting from structural vibrations. The sources of noise are labeled in the frequency spectrum of Figure 6. All three sources are important and each must be reduced to quiet the mower.

If the nonoccupational damage risk criteria of Figure 1 is plotted on Figure 6, the noise spectrum of the stock mower, it can be seen that a daily exposure of less than 30 minutes will not pose any damage risk whatsoever.

Mower noise levels at the operator for the typical lawn mower operations on grass were about 91 dBA. When the blade was balanced the level was reduced to 90 dBA. Satisfaction of the project goals requires a reduction of 5 dBA to 85 dBA at the operator.

CHAPTER IV

BLADE NOISE

Introduction

The noise of cutting grass has been discussed and it was found that this mode of operation did not radically change the mower noise spectrum, hence the remainder of this study will only be concerned with the aerodynamic noise of the blade, not cutting noise. From Figure 8 it is evident that the contribution of the blade noise to the noise of the complete mower is greatest at 300 and 400 Hz. Any effort to reduce blade noise must be concentrated on these frequencies. It is true that the A weighting network reduces the effective intensity of the low frequency noise and that reduction of the A-weighted level requires reduction of the high frequency noise, however, if one is concerned with reducing mower annoyance as well as the A-weighted level, the intensity of these low frequency tones must be considered. Reduction of the high frequency blade noise will not be useful until the other mower noise sources in that range have been quieted. The goals of the blade noise study are first to quiet the low frequency tones and to then explore solutions to the high frequency broadband noise.

In order to quiet specific peaks in the spectrum, their sources must first be clearly identified. These sources will be made fairly obvious after the theory of fan noise generation has been discussed. In addition to a discussion of previous work in mower blade noise, previous conclusions about propellor and fan noise control will be noted. Using some of the pertinent results of previous efforts, solutions to the blade noise problem will be tried and tested.

Identification of Noise Sources

Aerodynamic Noise

In general, aerodynamic fan noise consists of two important parts, periodic noise and broadband noise [24,25]. Periodic noise is tonal and occurs at integer multiples of the blade passage frequency as is indicated in equation (1).

$$f_n = nB\Omega \tag{1}$$

Periodic noise has two mechanisms of generation. It occurs as a result of the blade's rotating pressure profile and also because of localized disturbances of this rotating pressure field by fan struts or in this case, the grass deflector chute. Attempts to theoretically predict the magnitude of the sound power radiated from these two sources have been made by Gutin [24] and others, for some simplifed cases. The intensity of this noise is a function of the magnitude of the rotating pressure field, the number of blades, and their speed [24,26]. The intensity of the harmonics is a function of blade width, the wider the blade the less intense the higher harmonics [26].

Broadband noise appears in two major forms, vortex noise and turbulence noise. Vortex noise is the sound associated with the formation and shedding of vortices in flow past the blade and the related pressure fluctuations on the blade surface. It occurs as noise near the frequency

$$f = \frac{kv}{d} \qquad (2)$$

The intensity of vortex noise can be predicted for a few simplified cases of fans, propellors and rods. In general the acoustic power radiated is proportional to the tip velocity raised to the sixth power [24].

The other source of broadband noise, airstream turbulence, occurs when fan air enters relatively still air. Its magnitude and frequency are functions of air stream velocity and the amount of interference in the flow of air in and out of the fan [24].

The above discussion applies to the noise of propellors and fans and does not consider the effects of the shroud on blade noise. To apply the previous discussion to the blade, its fan noise was determined in the semireverberant room. Although an experiment of this type would be much more valuable if performed in an anechoic space, the study did point out some interesting results. Curve A of Figure 11 shows the frequency spectrum of the mower noise when the blade is powered by the electric motor. Curve D is the noise of the blade itself when motored in the open. The only noise expected in curve D is that due to the rotating pressure profile, vortex noise and turbulence. From equation (1), the rotation noise of a blade turning at 3000 rpm is expected at 100n Hz and is so noted in Figure 11. See Appendix D for calculations. Note that the harmonics of the rotation noise are very weak when the rotating profile is undisturbed.

Although vortex noise might be generated all along the blade, it is expected to be the most intense at the end of the blade since the blade tip has the greatest velocity. Using equation (2), the frequency of the tip vortex is found in Appendix D to be about 5000 Hz. The peak near 5000 Hz in Figure 11 is therefore assumed to be due to the blade tip vortex.

When the mower was operated with a surface near the blade, curve C of Figure 11 resulted. The broadband noise around 1000 Hz is assumed to be due to vortices leaving the trailing edge of the blade and airstream turbulence. It is thought that the addition of the surfaces around the blade reinforces this component of noise and results in the changes in the spectrum of Figure 11. It is not understood why the vortex noise at 4500 Hz did not increase when the mower

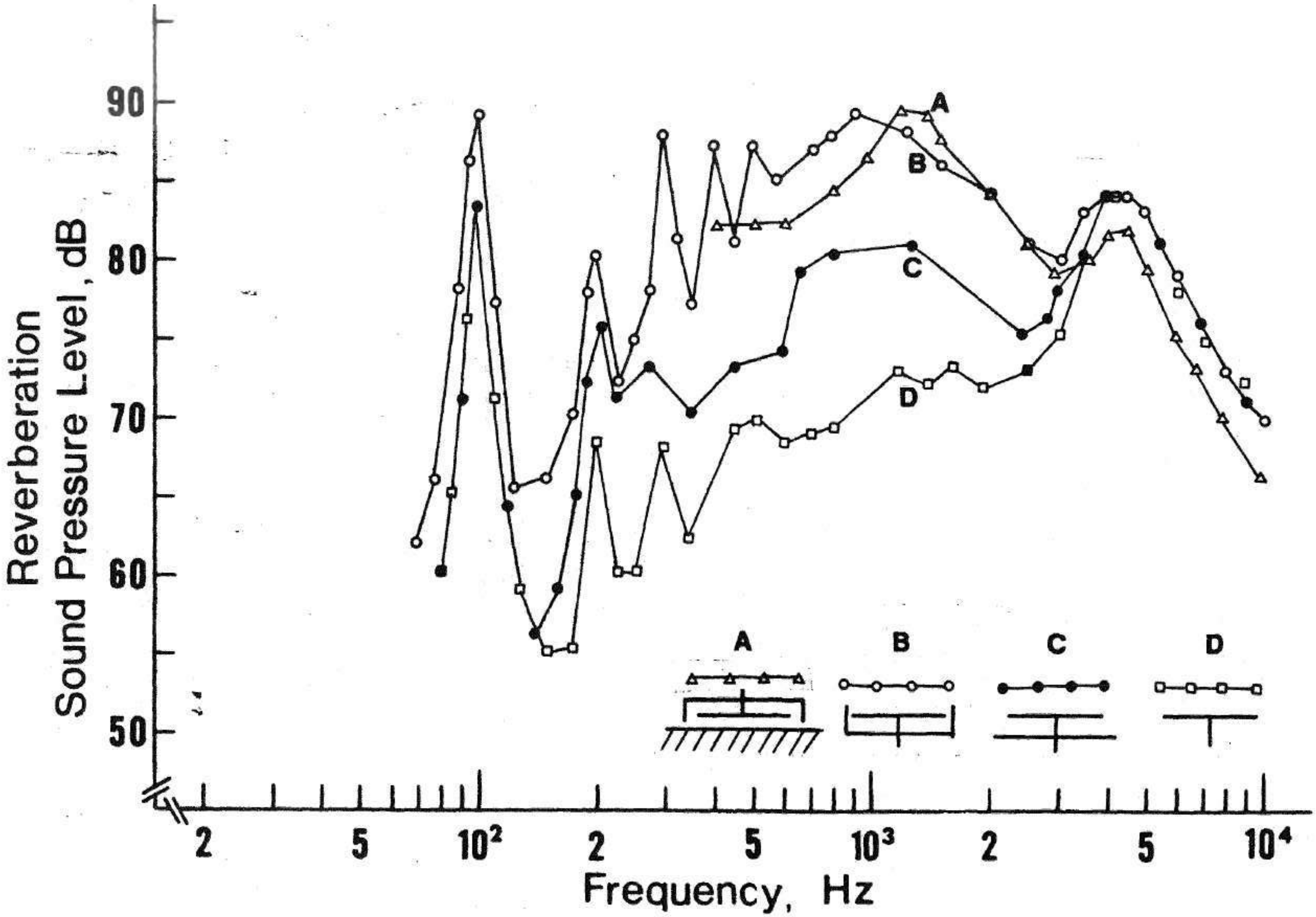


Figure 11. Blade Fan Noise

shroud was in place. When the shroud is in place the noise at the fundamental and all of the rotational harmonics increases. This is considered as being due to the disturbances of the blade's rotating profile by the grass deflector chute.

Nonaerodynamic Noise

With properly balanced machinery, the nonaerodynamic vibration noise of a fan itself is often insignificant [24]. If unbalanced, however, the fan may excite vibrations in itself, its supporting structure and in its drive train. Although there are many vibrational modes possible, only the vibration of the blade normal to the ground is considered. important. Cantilever vibrations may be important because blade bending modes might be excited as the rotating pressure profile is disturbed.

Using equation (3) which may be found in any vibrations text [28], bending mode frequency may be predicted.

$$f_n = \frac{1}{2\pi} a_n \sqrt{\frac{EIg}{\eta \ell^4}} \tag{3}$$

where

$a_1 = 3.52$

$a_2 = 22$

$a_3 = 61.7$

$I = \frac{hd^3}{12}$

The fundamental blade bending mode is found from equation (3), see Appendix D, to be 50.49 Hz. Experimental results with an accelerometer found the fundamental to be 47.6 Hz. From the experimental results, the radical term in equation (3) was corrected and the higher harmonics were calculated using this value. It is shown in Appendix D that the first three harmonics are at 47.6 Hz, 297 Hz, and 823 Hz. The only vibrational mode that might be excited is the second cantilever mode at 297 Hz. It was felt that this mode was unimportant since the noise spectra of other blades having different vibrational mode frequencies had nearly the same noise near 300 Hz.

On the basis of the previous discussion, the important sources of aerodynamic blade noises are labeled in Figure 12, the spectrum of the mower operating on asphalt when powered by the electric motor.

Previous Studies of Blade Noise Control

Propellor Noise

Regier and Hubbard [26] have summarized research on aircraft propellor noise and present the following conclusions:

(1) For a given power, the addition of more blades to a propellor always reduces the noise. This is due to the cancellations of harmonics in the low frequency periodic noise.

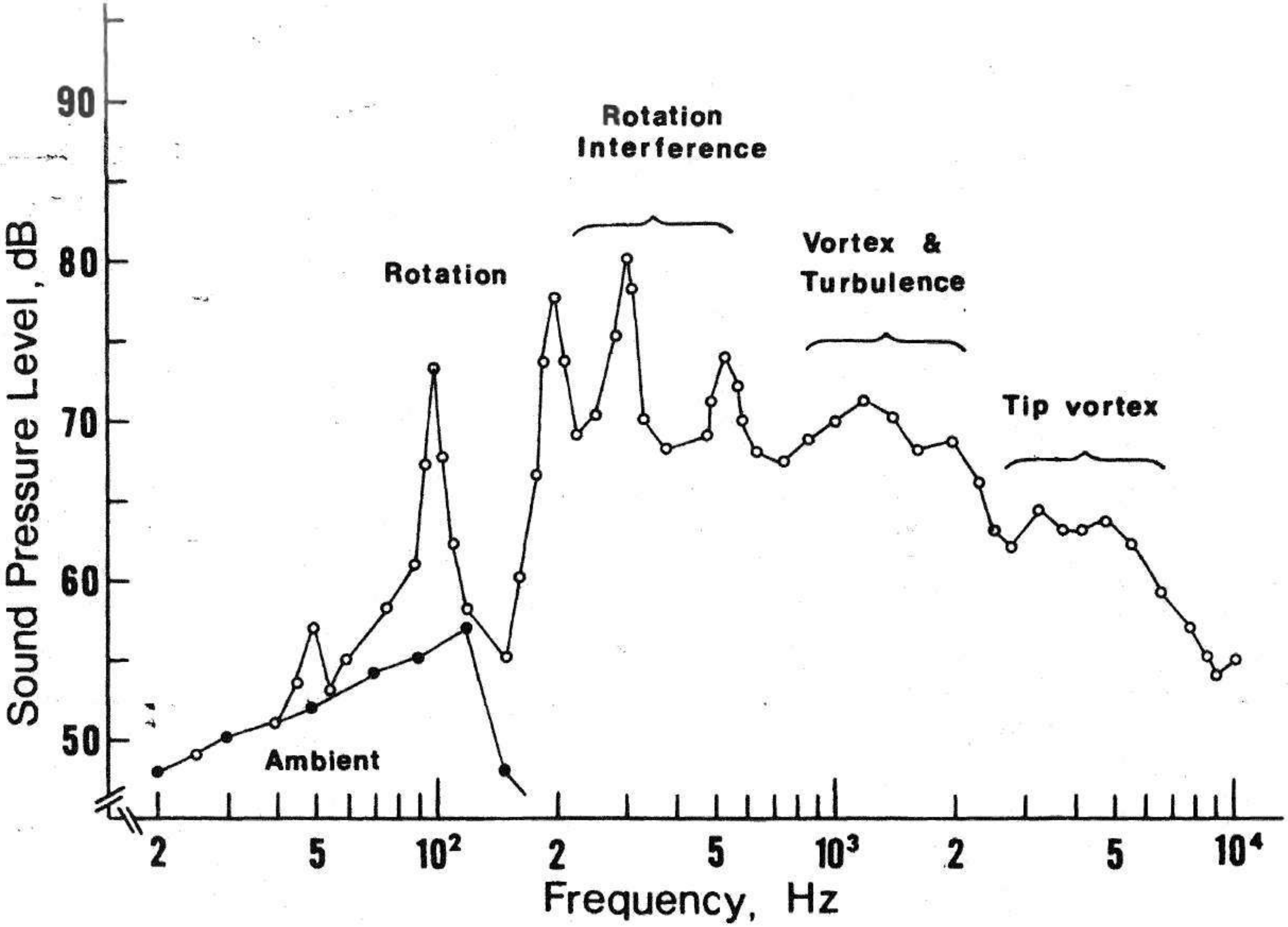


Figure 12. Sources of Blade Noise

(2) As blade width increases the intensity of the higher harmonics of the periodic noise decreases.

(3) As propellor speed decreases so does the noise.

(4) The value of a cowl as a noise reducing device is questionable.

(5) For many applications the requirements for a quiet propellor are compatible with propellor efficiency.

Fan Noise

In a report on truck fan noise, Filleul [27] presents the following conclusions:

(1) The addition of a cowl around the fan increased the noise 6 dB, the maximum increases occuring in the middle of the spectrum. More pronounced peaks at the blade passage frequency and its harmonics also resulted.

(2) The addition of a bell mouth on the inlet side of the cowl reduced the noise to the original levels showing that the noise with the cowl present was due to turbulence noise generated by air spilling over the cowl's edges.

(3) When a strut was placed 1 inch from the plane of rotation of a 12 blade fan, noise levels increased from 3 to 8 dB with the maximum increases occuring in the second and third harmonics. The presence of the strut had little effect on the noise of a 2 or 4 blade fan.

(4) When the number of blades was reduced from 12 to 2 the overall sound pressure level decreased only 4 dB.

Mower Blade Noise

Sperry and Sanders [2] have studied the noise of lawn mower blades in some detail. Octave band levels were used to determine how various blade parameters affected the noise. In their study, an 18 inch rotary mower blade was mounted in a reverberation chamber without a shroud and rotated at 3600 rpm. Among the parameters studied were blade width, condition of the edges, i.e. sharp or blunt, length of sharp edges, end conditions, hub radius, lift, and the noise of various 'S' shaped blades. Most of their concern was with the reduction of high frequency noise. They reached the following conclusions concerning noise in the range 600 Hz to 9600 Hz.

(1) Octave band levels decrease about 5 dB when the blade leading edge is sharp. Although levels decreased with the length of the sharp leading edge, no improvement was noticed after five inches of sharpening.

(2) Sharpening the trailing edge had a similar effect as did sharpening the leading edge except the 4800 Hz to 9600 Hz band, the band expected to contain the vortex noise, decreased 20 dB.

(3) As the blade width increased from 1-1/2 to 2-1/2 inches, octave band levels in the range 600 Hz to 9600 Hz increased about 5 dB.

(4) Changing the end condition of the blade, square, sloping, etc. (there is no concern here with blade lift) had

no effect on noise levels.

(5) As the blade hub size decreased from 8 to 4 inches in radius all levels decreased about 10 dB. Decreasing below 4 inch radius had little effect on blade noise.

(6) Bending the blade in an 'S' in the direction of rotation resulted in a 5 dB drop in octave band levels in the range 600 Hz to 9600 Hz.

(7) Fan type ends, i.e. blade with lift, had slightly less noise. More than 1/4 inch lift increased blade noise, indicating an optimum of 1/4 inch lift.

Summary

Previous efforts in the noise control of rotating machinery imply that the following variation of parameters will result in a quieter mower blade:

(1) Since the magnitude of the periodic noise is a function of the magnitude of the rotating pressure profile or the amount of lift, a blade with less lift should produce less intense periodic noise. It should be recalled, however, that this may have adverse effects on the operation of the mower especially when bagging grass.

(2) The effects of multiple blades is unclear, although with more blades equivalent lift might be generated with less noise.

(3) The removal of obstructions in the blade's rotating profile such as the grass deflector chute may reduce noise.

(4) Sharpening or streamlining the blade should produce a blade with less high frequency noise.

(5) Although blade noise will decrease with rpm, the speed requirements for acceptable cutting and grass dispersion render slowing the blade an unacceptable solution. All testing will be done at 3000 rpm.

Results

The first solution tried was that of reducing lift. The stock blade, which will be referred to as blade 1, had a lift of about 1/2 inch and was replaced with an otherwise identical blade, blade 2, with 1/4 inch lift. The result was a 3 dB decrease to 87 dBSPL. The A-weighted level remained the same at 85 dBA. The two spectra are shown in Figure 13. From the spectra, one can see that the periodic noise has been reduced about 4 dB. The high frequency noise was unaffected. All further tests will be made with blade 2, the one with less lift.

When the trailing edge of blade 2 was sharpened it had absolutely no effect on blade noise as may be seen in Figure 14. The difference in noise between a sharp leading edge and a dull one was not investigated. The effects of removing part of the grass deflector chute from the mower can be seen in Figure 15. The A-weighted level was unchanged and the SPL dropped 1/2 dB as a result of the expected decrease in the periodic noise.

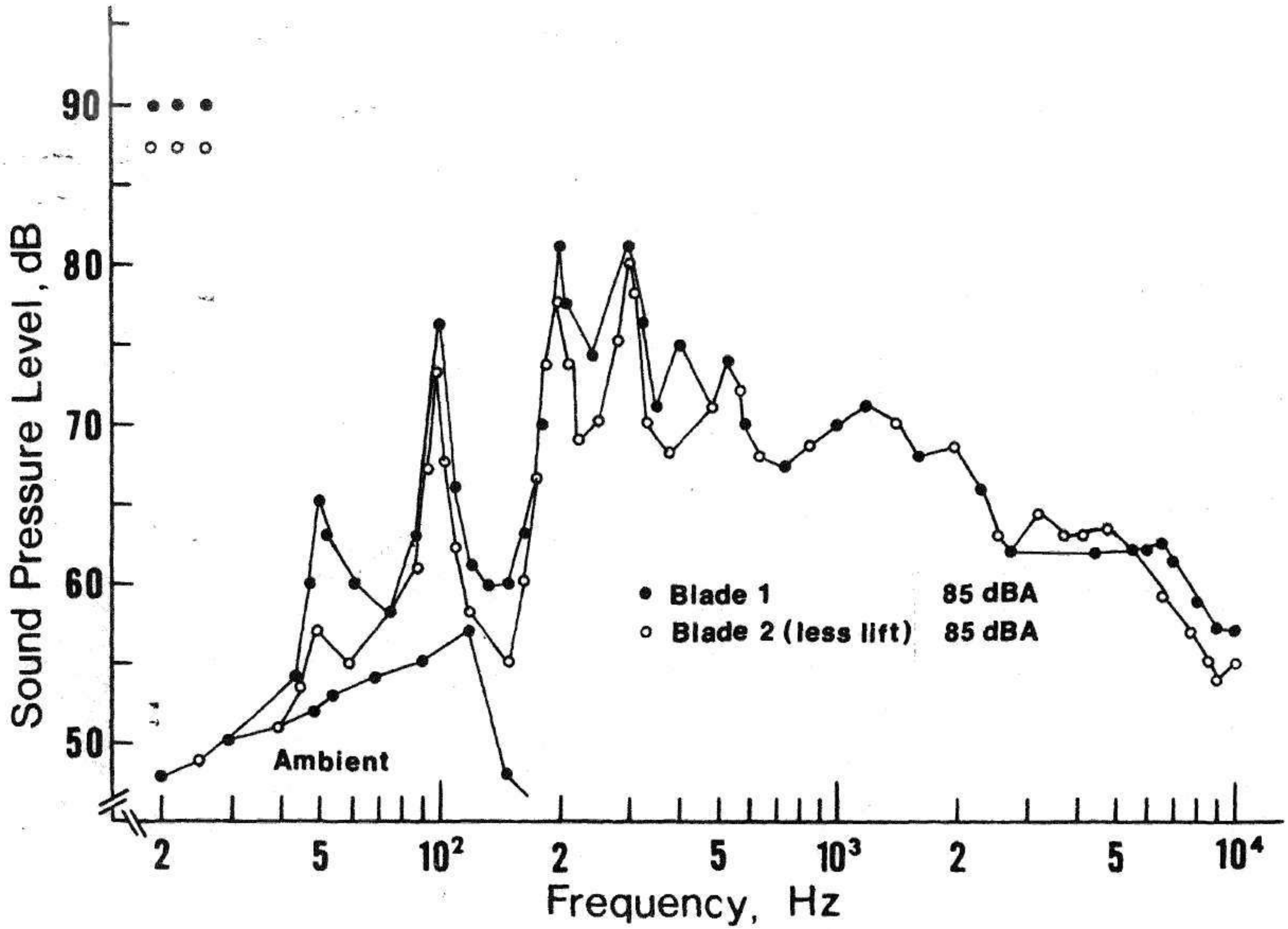


Figure 13. Effect of Lift on Blade Noise

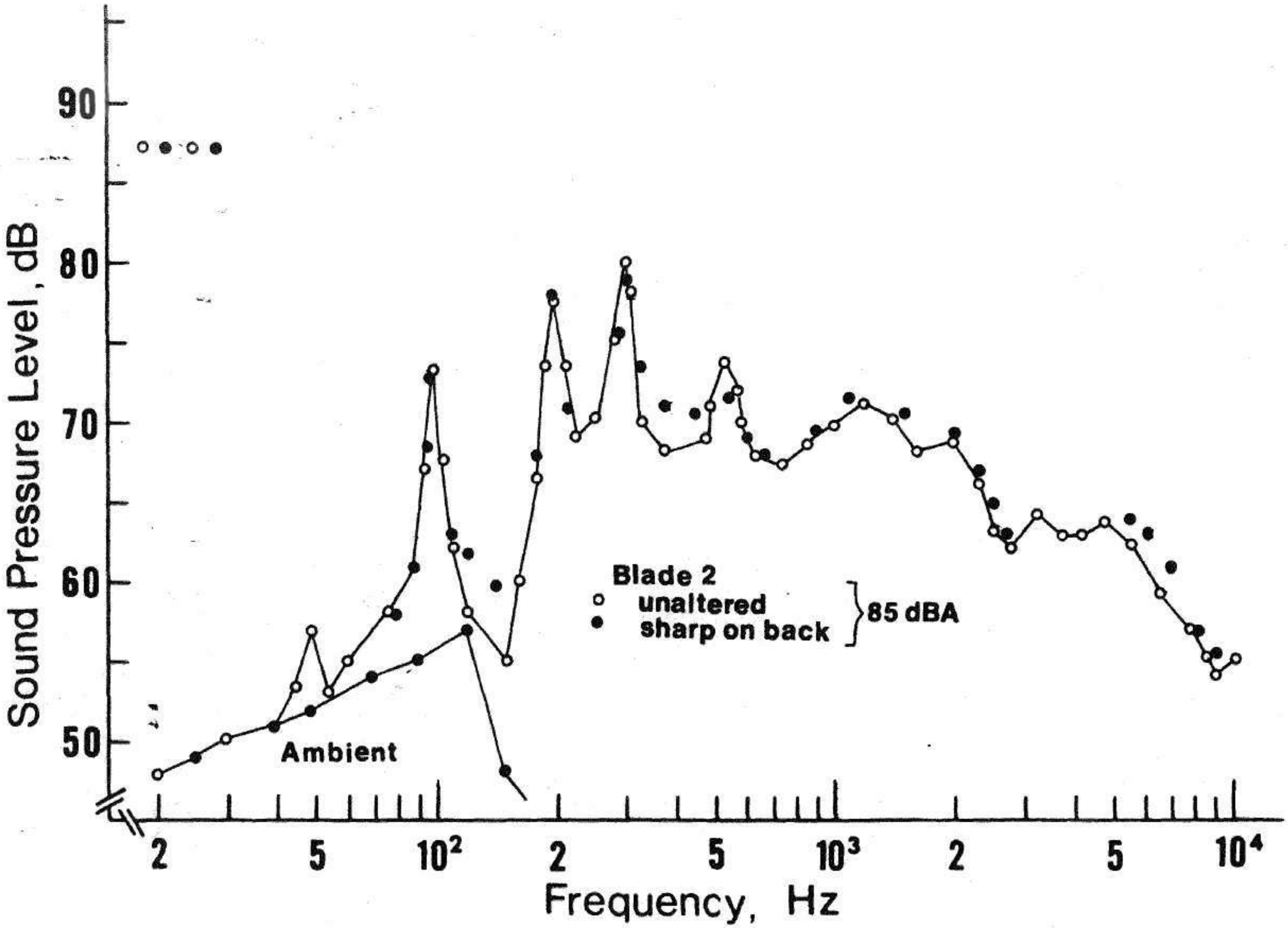


Figure 14. Effect of Sharp Trailing Edge on Blade Noise

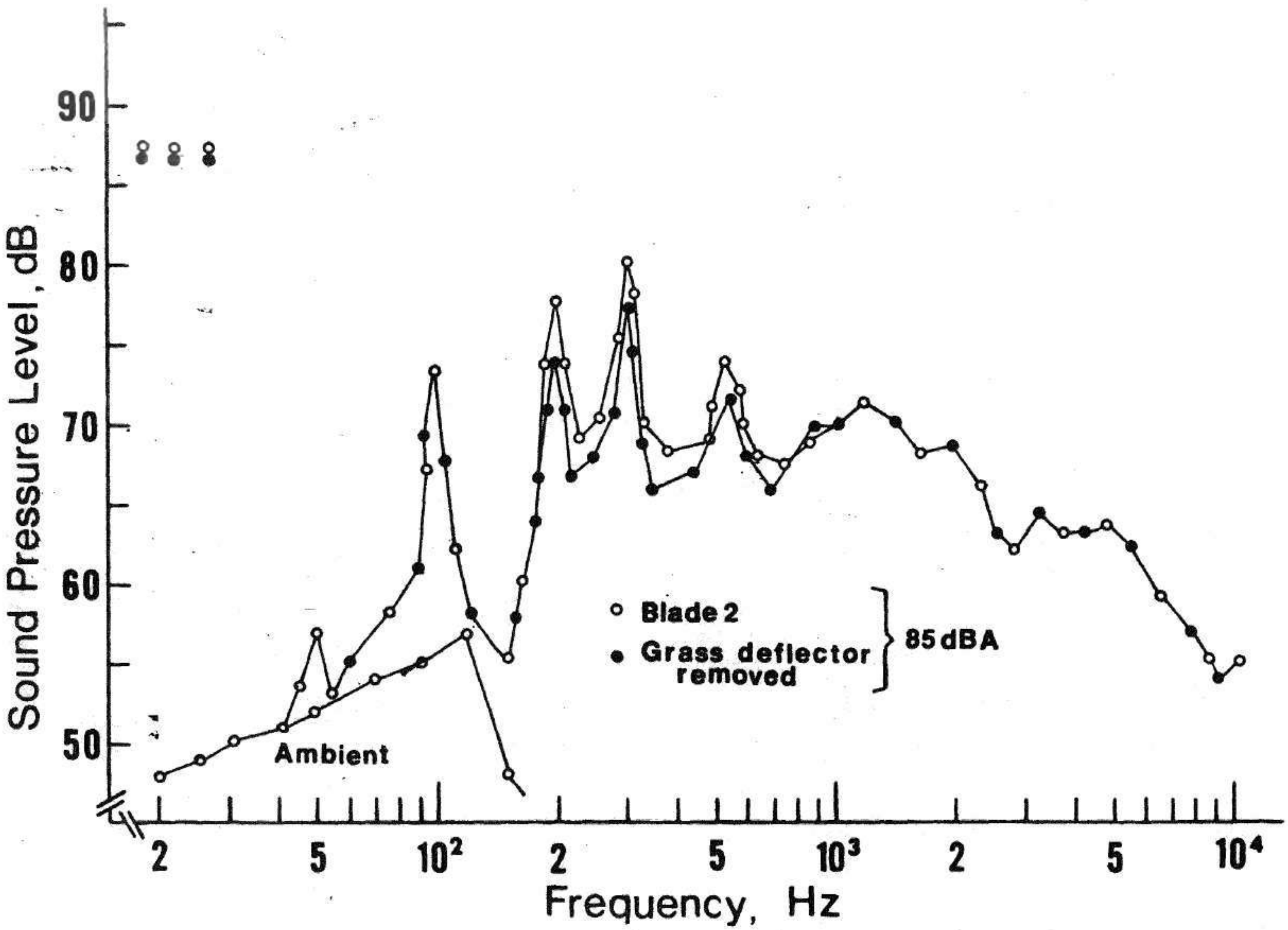


Figure 15. Effect of Grass Chute on Blade Noise

Figure 16 shows the spectra of the mower operating with two blades similar to blade 2 which were bolted on 90° apart. Also plotted is the spectra for blade 2. Overall levels increased over those for one blade as can be seen in Figure 16. Reiger and Hubbard's first conclusion stipulated a given power into the propellor. Their conclusions imply that noise decreased as the number of blades increases and lift remains constant. A more valid judgment of the four-ended blade might be made if it were compared to a two-ended blade of equivalent lift. Although it is not known if their lifts are equivalent, Figure 17 matches the noise of the four blade fan with that of blade 1. It can be seen that the noise levels of the two blades is about the same.

In addition to modifications of the standard blades, two other blades were tested. One, a Toro blade, was bent in an 'S' shape, but interestingly enough it was bent opposite to the direction recommended by Sperry and Sanders. It had the same diameter and lift as blade 2. Its spectra was quite similar to that of blade 2. Although the SPL was 87 dB and the A-weighted level was 83-1/2 dBA, no reason for this decrease is given here. The other blade tested was similar to blade 2 except it was 1/2 inch shorter. The spectra for this blade and blade 2 is shown in Figure 18. The levels were 86 dBSPL and 83 dBA.

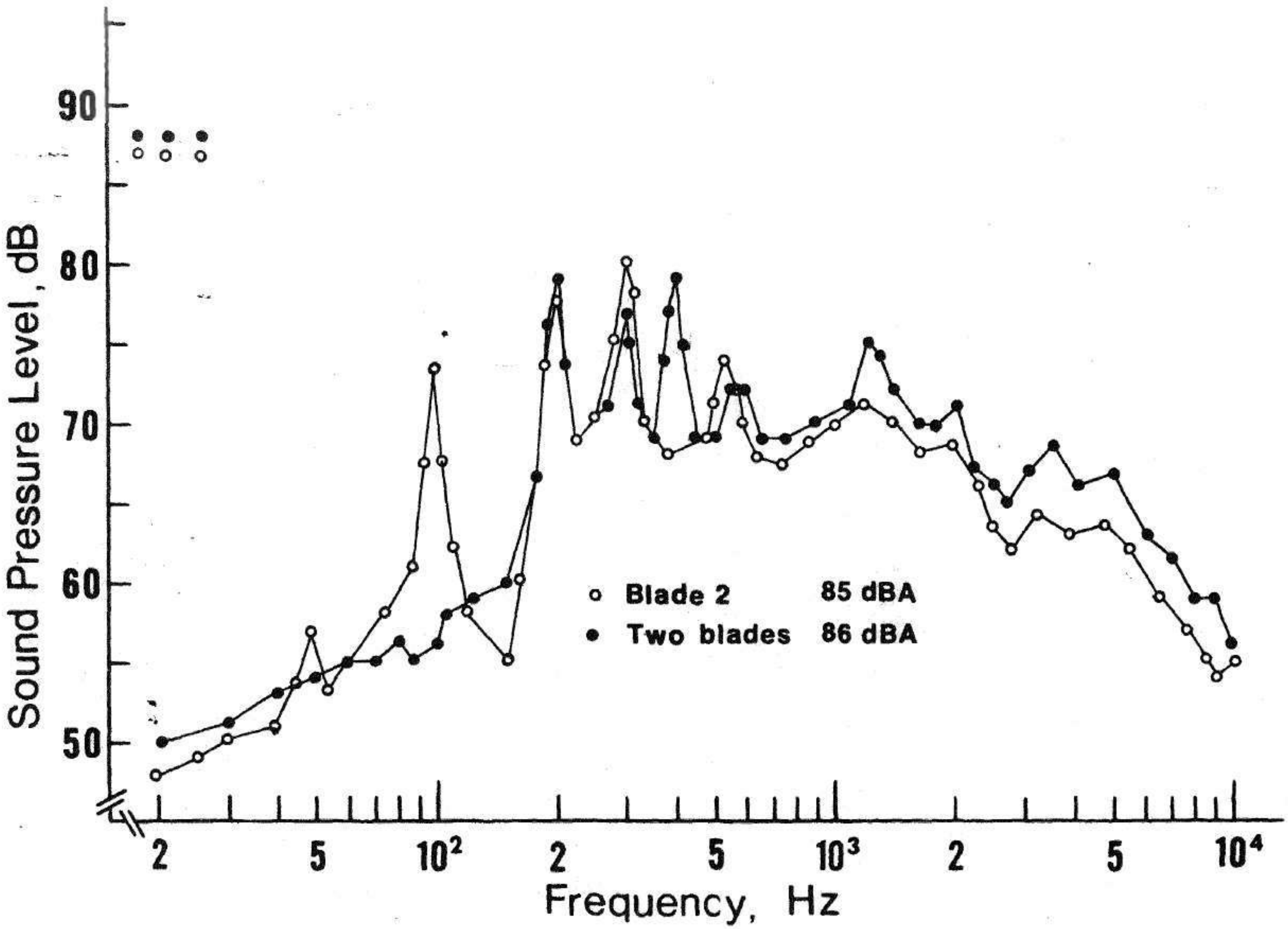


Figure 16. Effect of Additional Blades on Noise

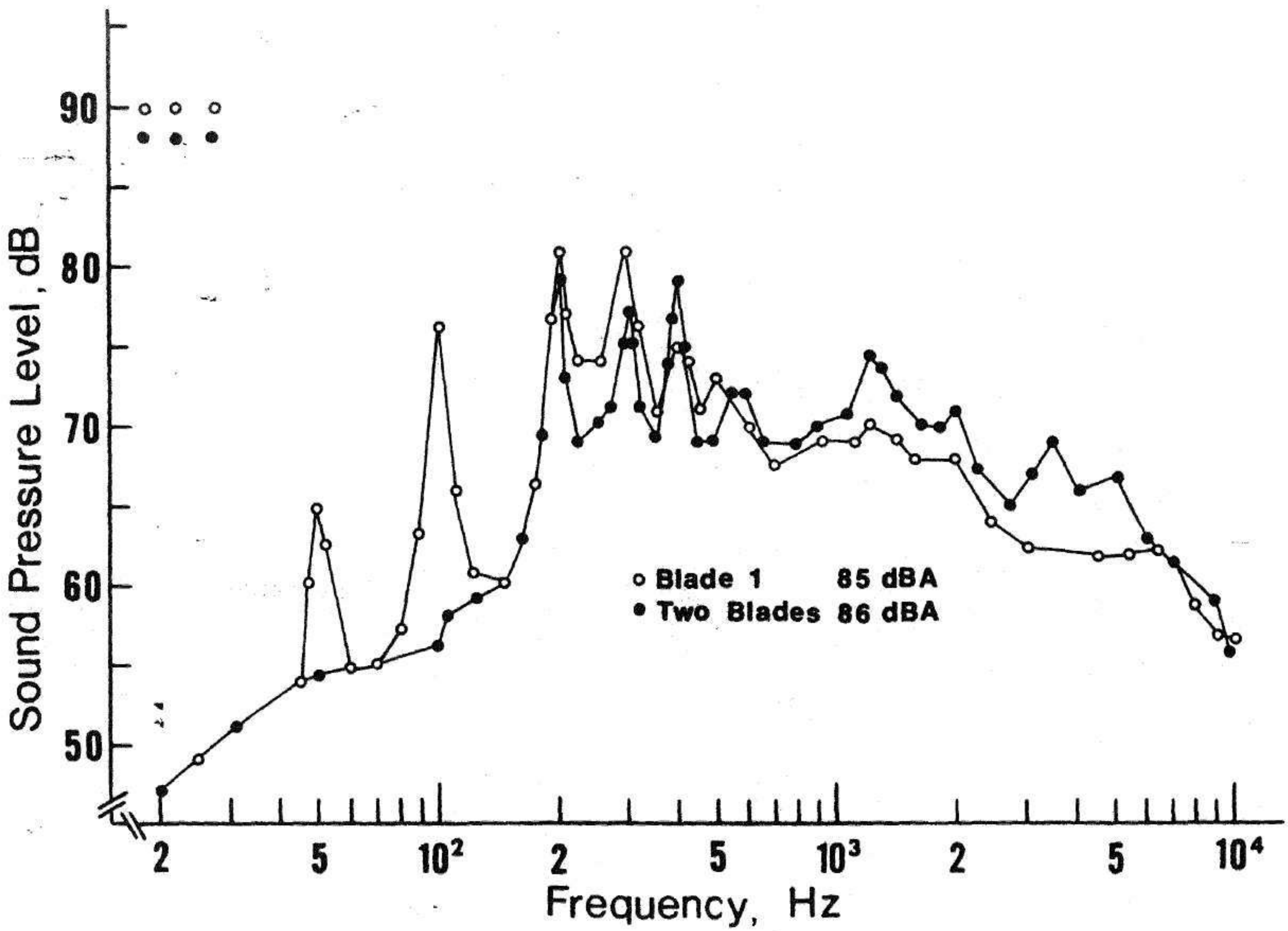


Figure 17. Effect of Additional Blades on Noise

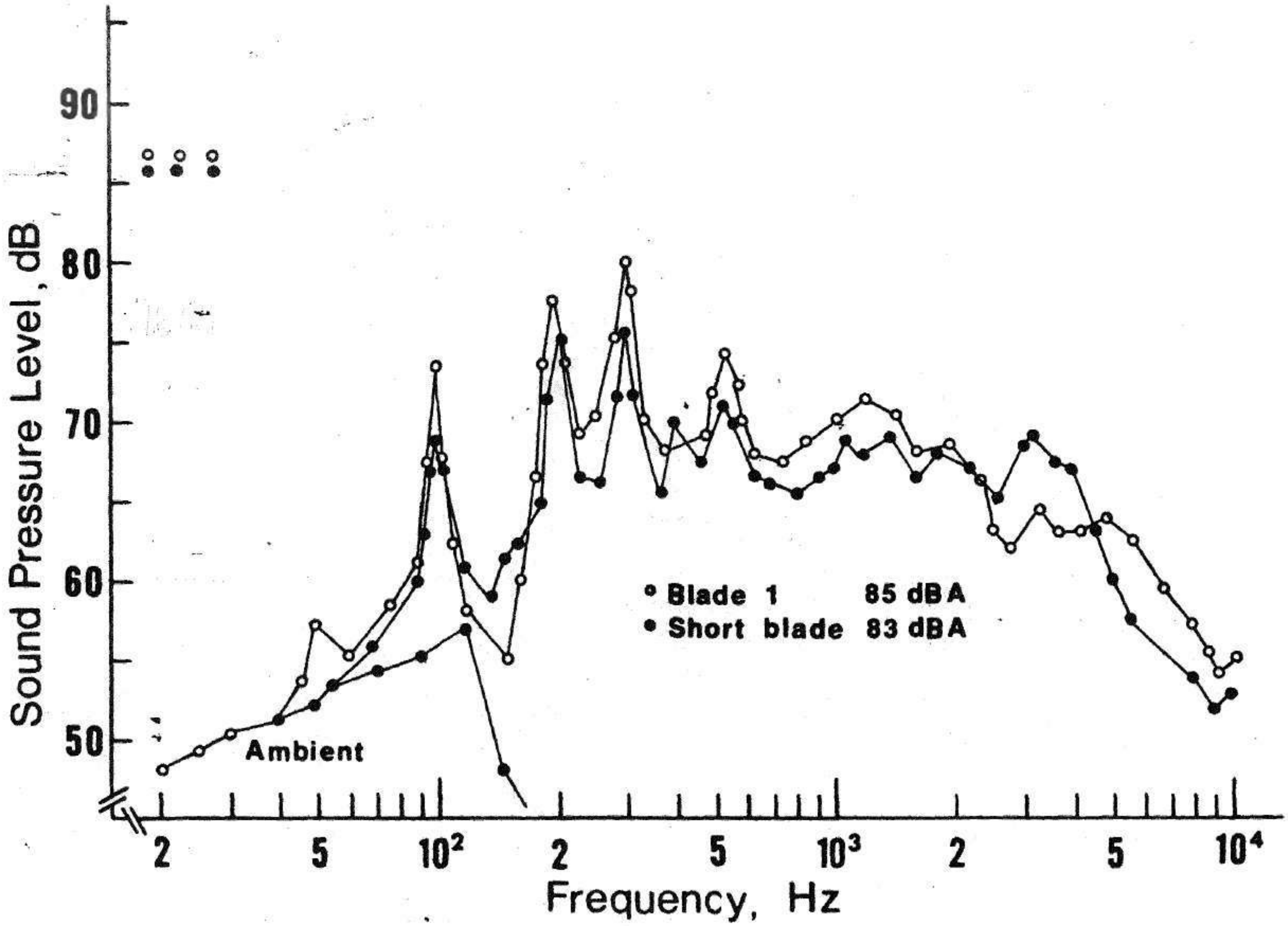


Figure 18. Effect of Length on Blade Noise

Conclusions and Recommendations

The major components of blade noise are periodic rotation noise and broadband noise resulting from vortices leaving the blade. The mechanism of generation and amplification of the midrange broadband noise is not clearly understood. Although some previous results concerning the noise of propellors and fans may be extended to blade noise, specifically in the area of rotation noise, these results do little to explain the mower blade's turbulent noise. Further study of the mechanism of lawn mower blade noise generation, particularly the role of the shroud, is necessary before this noise will be completely understood. Guenther of Ohio State University is currently researching this problem.

The results of the blade noise study are summarized in Table 4.

Table 4. Blade Noise Levels

Blade	Noise Level dBA	Noise Level SPL
Blade 1	85	90
Blade 2 (less left)	85	87
Blade 2 sharp on trailing edge	85	87
Blade 2 without deflector shute	85	86½
Two blades like Blade 2	86	88
'S' shaped blade	83½	87
Short blade	83	86

Of the parameters studied in this project, only the reduction of blade lift was significant in reducing the overall sound pressure level. None of the parameters studied resulted in a decrease in A-weighted level. Sharpening the trailing edge of the blade had no effect on reducing blade noise. With a properly designed multibladed cutter it may be possible to reduce noise and still maintain the necessary blade speed and lift.

Since the shorter and 'S' shaped blades were quieter, improvement in blade noise is obviously possible. Parameters that might be studied include the distance between the blade and the shroud, and the clearance between the blade tip and shroud skirt. More specific recommendations on the optimum blade lift are also necessary. Hopefully, with a more complete understanding of the mechanism of lawn mower blade noise generation will come intelligent design recommendations that will result in quieter blades.

CHAPTER V

VIBRATION NOISE

Introduction

Examination of Figure 7 reveals that the source of acoustic vibrations at frequencies above 1000 Hz is blade noise or structural radiation. Figure 8 implies that the dominant noise at frequencies above 500 Hz is structural vibration rather than blade noise. Low frequency vibration does not seem to be an important source of radiated noise. This is probably the result of the inability of a body to effectively radiate sound at frequencies having a half wavelength greater than the largest dimension of the body. The primary effort in the vibration noise study is to reduce the vibration induced noise in the range 500 to 10,000 Hz.

Sources and Possible Solutions

Most of the vibration induced noise results from one of three sources, radiation from the engine, radiation from the blade enclosure, and miscellaneous rattles specific to a particular model of mower.

Rattles

Most of the noise in this group such as a loose gas tank or starter mechanism is the result of poor design and maintenance. Faulkner [8] considered the noise of a riding mower resulting from loose and vibrating parts to be significant. He found a 3 dBA reduction possible by placing his hand on a vibrating fender. Although most mowers have loose and vibrating parts which cause noise, these sources are unique to each particular mower and are not of general interest. They were not a significant contribution to the overall levels of the mower being studied and only one such source will be discussed.

Mower safety regulations require a shield that drags on the ground between the rear wheels to prevent articles from being thrown out in the direction of the operator and to keep the operator's feet from getting underneath the mower. The shield on the mower being studied was metal and resulted in a noise level at the operator of 85 dBA when the mower was pushed on a sidewalk with the engine shut off. The manufacturer has since corrected this by installing rubber skids on the shields of more recent models.

Engine Noise

In studying riding lawn mower noise, Faulkner [8] concluded that direct radiation from the engine casing itself was the dominant noise of the system. Noise from small engines has been examined by Kamo and Iwatsuki [29] of Armour Research Foundation. It is their conclusion that mechanical engine noise originates as the vibration of structural members, the crankshaft, camshaft, valves, gears and bearings. Unbalance of fluctuating forces in these elements excites the engine's structure which vibrates, hence radiating sound. The intensity of the radiated sound depends on the mass of the body, its stiffness, its damping and its dimensions. If the body is heavy and limp it will be a poor radiator of high frequency sound [12].

To reduce the noise caused by structural vibration, the following solutions which will reduce the noise at the source are possible. First eliminate the excitation force. This could be done in part by dynamic balancing of the engine. Falkner [8] states that this would result in an overall noise level reduction of about 4 dB at a cost of $12. This would not completely eliminate the excitation, however, since there would be torsional fluctuations in the valve train and in the crankshaft. Some of the later vibrations might be reduced by using a heavier flywheel which might reduce fluctuations in rpm. An obvious improvement here would be the use of a rotary engine.

A second solution might be the use of engine material with high damping and heavy mass such as cast iron. Current trends toward aluminum engines run counter to both of these objectives, aluminum being light and having little damping. A third solution might be the reduction of the area of radiating surfaces by avoiding large flat panels, such as on the gas tank, wherever possible.

The above solutions are not solutions at all, but are important design constraints if noise is to be a consideration in the development of an engine. If noise was not a constraint in the design of an engine and it turned out to be noisy, the next alternative would be to enclose it. This is one approach taken by the manufacturer of a popular garden tractor at an increase in cost of more than \$100 on an \$1800 machine. An enclosure can provide excellent noise reduction if it is nearly airtight, but unfortunately, air cooled engines require that cooling air be circulated around the engine, meaning that the enclosure must have holes in it. In order for an enclosure with holes in it to be effective, the air passing through the holes must also pass through some type of acoustic filter or baffles that will not let out the acoustic energy inside the enclosure.

One can appreciate that abatement of the structurally radiated noise from a conventional internal combustion engine is no simple task. Since there was no means available in this project of easily determining the contribution of the engine noise, the benefits of reducing this portion of the noise were not known. The costs of reducing engine noise were known to be excessive. For this reason it was decided to concentrate on structural radiation from the shroud.

Radiation from Blade Enclosure

Nothing was said in the previous discussion about isolating engine vibrations from the rest of the system. The important vibrational modes that originate in the engine or blade and may be transferred to the shroud (see Figure 19)

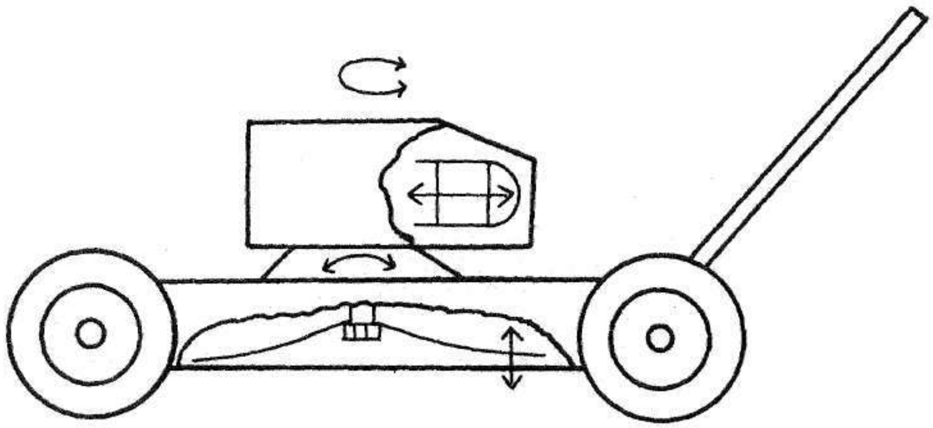


Figure 19. Lawn Mower Vibrational Modes

are shaking in a horizontal plane due to the motion of the piston, torsional vibration about the crankshaft as a result of the engine varying torque, and shaking in a vertical plane due to possible dynamic unbalance of the blade. The oscillation of the piston produces an additional rocking moment of the engine on the shroud. The force transmitted to the shroud as a result of this rocking moment can be minimized by proper placement of the engine mounts.

What effects do these forces have on the radiation of sound? Sound will be radiated most efficiently from a large panel that responds to some driving force. Obviously the most important panel is the top of the blade enclosure, being driven by the rocking moment of the piston oscillations and possibly the blade. The torsional and horizontal forces may certainly be important sources of vibration, but little sound will result unless they excite an effective radiator.

It is apparent that the most important vibration isolation will be that which isolates the shroud from vertical engine vibrations. By probing with an accelerometer it was found that when the mower was running the maximum accelerations on the shroud were normal vibrations of the top surface.

The previous discussion leads to the system model shown in Figure 20 and the problem of isolating the engine such that high frequency vibrations, those above 500 Hz, will not be transmitted to the shroud. It should be made clear that the exciting force is not a sinusoidal vibration at 25 Hz, but is instead broadband noise consisting of almost all frequencies in the audible range.

Although the concept of mobility [12] provides a method of accurately predicting the effects of an isolator, without instrumentation to determine the mobility of a complex structure it can provide little more information than classical vibration theory. Creed [30] considers the model

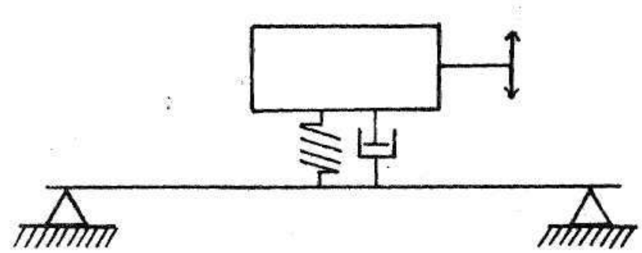


Figure 20. Simplified Model of Lawn Mower

shown in Figure 20 acceptable for a machine mounted on an elastic foundation when the half wavelength of the forcing function is greater than the thickness of the isolator. When the half wavelength of the forcing function is small relative to the thickness of the isolator, the model in Figure 20 is not acceptable. In the latter case the phase change of the wave in the mount must be considered. For reasons to be made clear soon, the mount used will be made of rubber. At 2000 Hz, the wavelength of sound in a typical rubber is about one inch [30], hence the half wavelength will be about one half inch. Frequencies above 2000 Hz will have shorter wavelengths. Structural constraints require that the mount be as thin as possible, hence it is evident that both cases of vibration must be considered.

Using classical vibration theory to describe the model in Figure 20, the following simplifying assumption will be made. The mass of the isolator is negligible. This is valid since it weighs less than a pound, the engine and blade weighs 24 pounds and the base weighs 28 pounds. The damping in the isolator is negligible, the loss factor or damping for even high damping rubber being very small. The effect of the damping is not important except at resonance, however, the mount is being designed for frequencies well above resonance where its effects would produce little changes in the results. The next assumption is that the damping in the panel is viscous. This is probably acceptable as a first approximation. It can be shown, see Appendix E, for the given assumptions, that the system behaves according to Figure 21.

To reduce the vibrations of the panel for some force excitation of the mass, the following conclusions may be drawn from Figure 21:

(1) Softening the isolator decreases Ω_{x1}, increases $\frac{\omega}{\Omega_{x2}}$ and decreases $\frac{\Omega_{x2}}{\Omega_{x1}}$ resulting in lower values of $\frac{x_1}{F/k_1}$.

(2) Increasing the rigidity of the panel increases Ω_{x1} and decreases $\frac{\Omega_{x2}}{\Omega_{x1}}$ moving the peaks, places where ω and Ω_{x2} coincide, to a region where the isolator is more effective.

(3) Increasing the mass of the machine will reduce the natural frequency of the isolator and Ω_{x2}, decreasing transmissibility.

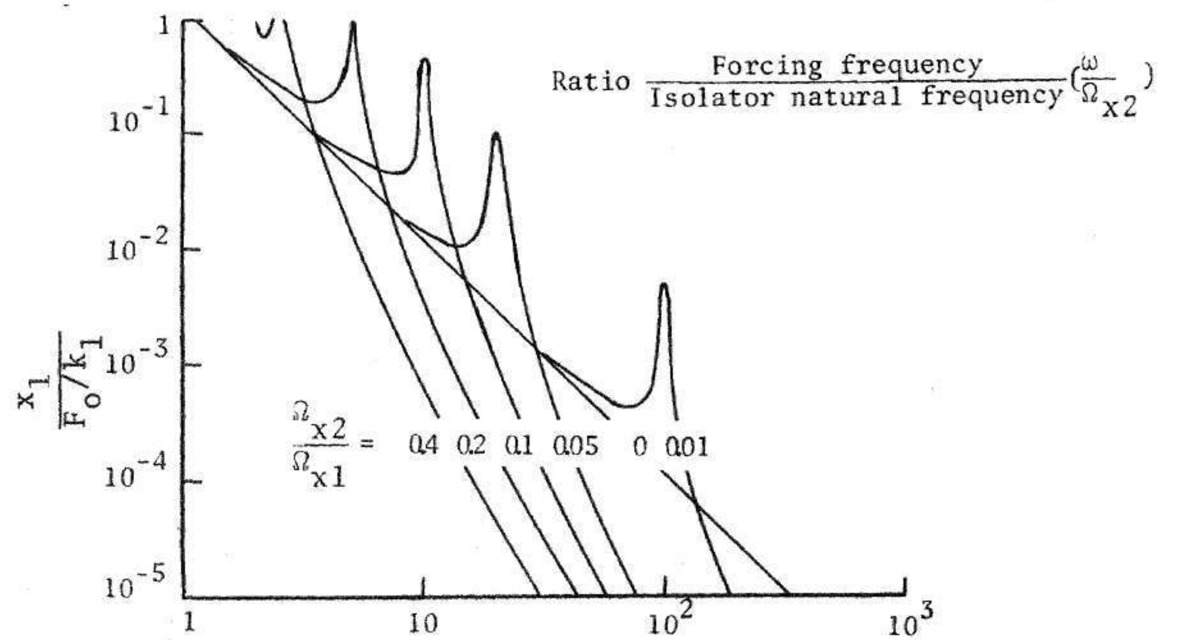


Figure 21. Transmissibility Curves for Mower Vibrations

When the wavelength is short and the assumptions of the classical theory are invalid, the motion of the sound wave in the material must be considered. An in depth analysis of this subject will not be considered here, but the conclusions are identical to those mentioned above with one addition. The concept of impedance will be discussed in more detail in the next chapter, where it will be shown that if the acoustic impedance, the product of material density and velocity of sound in the medium, of two adjacent materials is different, there will be an acoustic reflection at the boundary. For example the traveling wave in a vibrating clothesline will return when it reaches the end because there is an impedance mismatch between the clothesline and the supporting pole. With this in mind it can be seen that a material made of something other than aluminum, such as rubber, would be a more effective isolator at high frequencies.

Summary

The constraints for the isolator may now be clearly stated.

(1) The isolator should be as soft as possible.

(2) Since the engine is aluminum the isolator should not be aluminum.

Results and Conclusions

The previous discussion implies that mounting the engine on very light mounts would be the best solution.

This would be true were it not for the requirements in the Outdoor Power Equipment Institute's regulations. For certification by that group, the mower must pass a fairly torturous structural test. For this reason, the solution decided on was a rubber spacer to fit between the engine and the shroud. The rubber used was a special isolator material with a honeycomb type surface which reduces its stiffness in compression. The washer was simply bolted into the system, the heads of the bolts having washers made of the same material. Although no attempt was made to insulate the bolts from the holes in the shroud, thereby providing a path for the sound, the strength offered by this type of mounting offsets the benefits of reducing the lateral vibration.

Figure 22 shows the effects of the washer on mower vibrations. Data was taken with an accelerometer and analyzed with the wave analyzer. The shroud and engine base was probed with the accelerometer and no modal pattern was observed, thus it is felt that data taken at a point is representative of all of the shroud.

Figure 23 shows the change in mower noise levels when the washer was added. Overall levels remained the same, but the A-weighted level dropped 2 dBA.

Figure 24 shows the attenuation in noise levels and vibration levels that the washer produced. Note that in general, where the vibration levels decreased, so did noise.

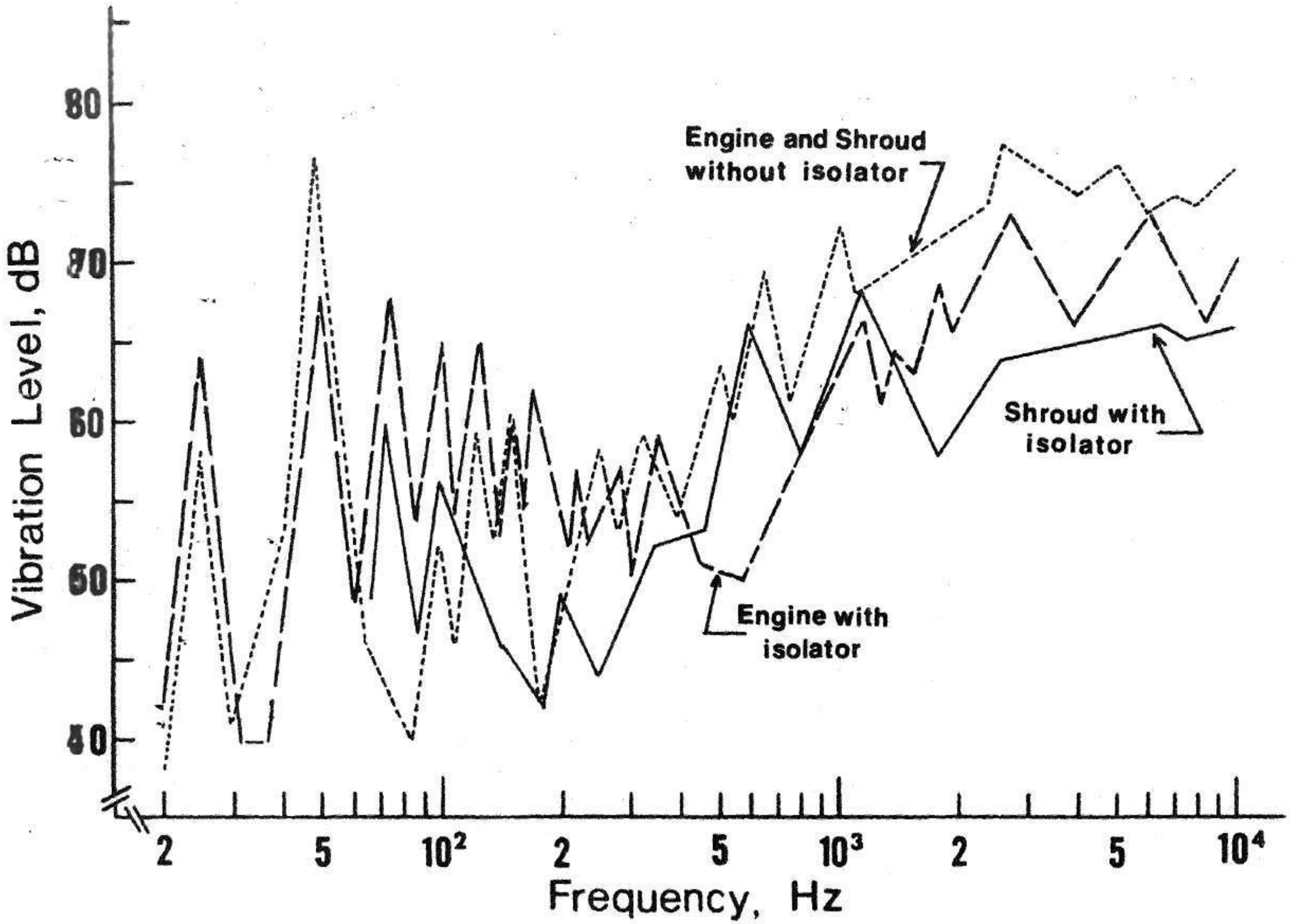


Figure 22. Effects of Isolator on Vibrations

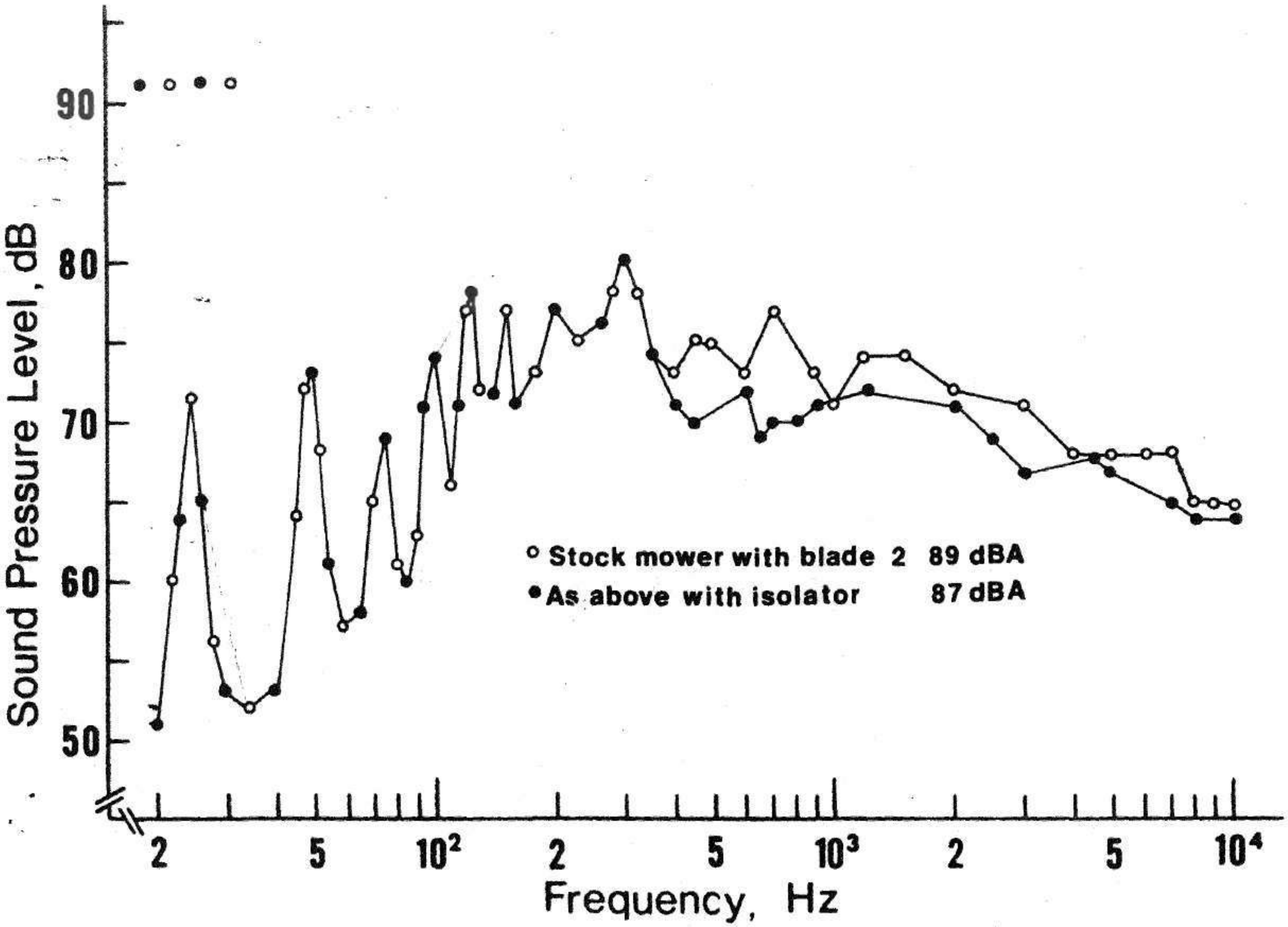


Figure 23. Effects of Isolator on Mower Noise

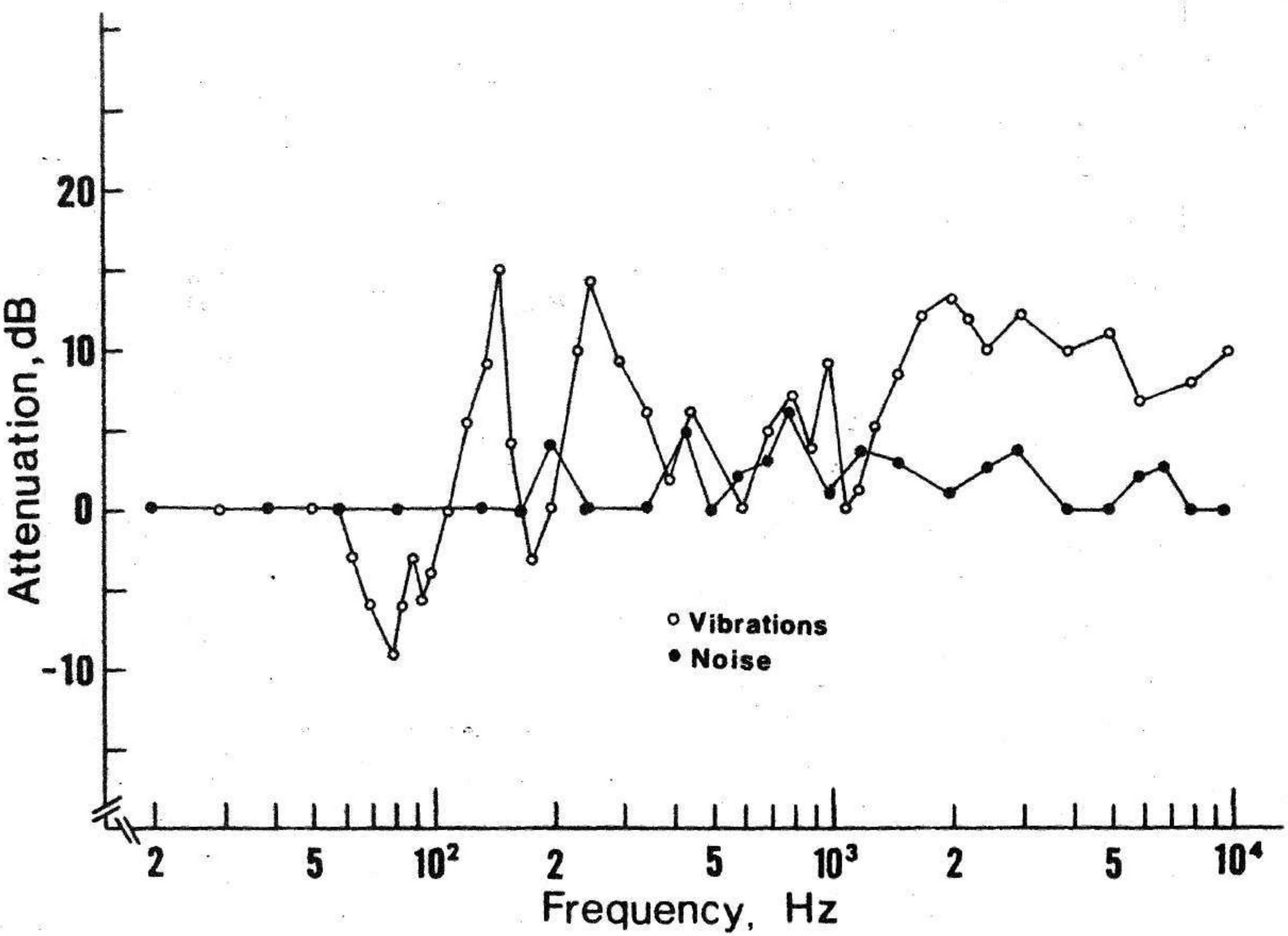


Figure 24. Attenuation of Noise and Vibration from Isolator

CHAPTER VI

EXHAUST NOISE

Introduction

Exhaust noise occurs as a result of the release of high temperature, high pressure gases from the combustion chamber and their resultant acoustic radiation into the atmosphere. Muffler systems are devices which either reduce the intensity of the acoustic perturbations radiated to the atmosphere or reduce the ability of the system to effectively radiate.* Figure 25 is a block diagram of an engine exhaust system. The source is the engine with its combustion process, valve openings and exhaust port. Most exhaust noise of single cylinder engines occurs as low frequency tones at integral multiples of the firing frequency. The amplitude of these tones is a function of load and rpm [24]. In addition to the

Figure 25. Block Diagram of Exhaust System

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*Some exhaust systems, especially those of two cycle engines, have the additional role of increasing engine efficiency via scavenging. The interested reader is directed to Obert, reference 31, for an excellent discussion of this subject.

low frequency noise there can be some higher frequency turbulent noise resulting from the gas flow [24].

The tube-filter-tube combination could be considered the muffler. It is in this portion of the system that the intensity of the acoustic perturbation is reduced. There are two major types of exhaust mufflers, dissipative and reactive. Dissipative elements absorb acoustic energy and dissipate it as heat when sound passes through or is incident upon an absorbent material in the element such as fiberglass wool. To be effective the thickness of the absorbent material must be at least one half of a wavelength thick, meaning that the dissipative element is most effective at high frequencies. Reactive elements either cause sound to be reflected back toward the source or cause the acoustic pressures of a system to interfere with each other causing pressure cancellations. Reactive elements are most effective at low frequencies, below 700 Hz, as will be shown later.

Examination of Figure 7 shows that exhaust noise is important below 1000 Hz, but most important below 300 Hz. For this reason reactive type mufflers will be discussed more extensively in this study than dissipative systems.

The termination of the exhaust system is extremely important. Although much research has been done in the past on muffler design, surprisingly little effort has been made to reduce the acoustic radiation efficiency at the tailpipe termination. This subject will be discussed in some detail in this chapter.

The purpose of this portion of the study is to reduce the exhaust noise of the lawn mower. This will be done by examining the currently available mufflers in search of a solution. After this search proves unsuccessful the theories and models available in the literature from which an intelligent muffler design may be made will be examined and the best theory chosen to design an acceptable muffler. Constraints for the muffler design will be based on the preliminary results and the project objectives. The radiation of sound out of the muffler system will be researched in the literature and an acceptable muffler termination will be designed. A system will be constructed and tested on the mower.

Description of the Physical System

The following points should be kept in mind while reviewing various methods of modeling the engine exhaust. The mower powerplant is a 10 cubic inch displacement, single cylinder, four cycle, internal combustion engine. Preliminary measurements made with a cromel-alumel thermocouple at an engine speed of 3000 rpm showed that the temperature in the exhaust port was about 1200°F, 1000°F in the muffler chamber and 900°F at the muffler exit. These changes in system temperature will result in a change of acoustic velocity in the exhaust system from 1940 ft per second to 1800 ft per second. Exhaust pressures were measured with the microphone probe, see Appendix F, and were found to be in the range of 150 to

160 dB or about .1 psi. The approximate shape of the exhaust pulse was observed on the oscilloscope using both the microphone probe in the muffler and a microphone near the open exhaust port. Both observations showed a pulse as shown in Figure 26. The important point to be noted in the figure is that exhaust noise is not a sinusoid.

Calculations indicate the average flow velocities through the muffler to be about 10,000 feet per minute which at the temperatures considered is equivalent to a Mach number of .1. In addition to these facts, it should be recalled that any sytem designed for the mower should be inexpensive, have low backpressure, and be small. Size is an important consideration in the selection of a proper exhaust theory.

<u>Measures of Muffler Effectiveness</u>

There are a number of methods of specifying the effectiveness of a muffler design. Transmission loss relates

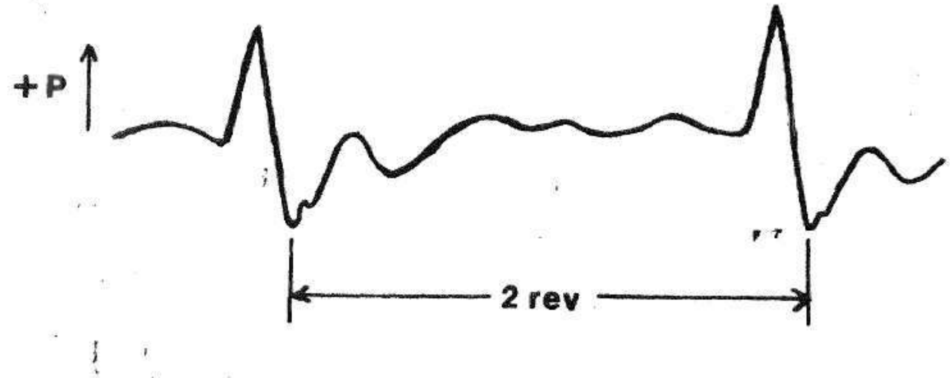


Figure 26. Engine Exhaust Pulse

the sound power incident on a muffler to the sound power transmitted. It is of little use in engineering since there is no acoustic wattmeter; most sound measuring devices respond to sound pressure. A more useful concept is attenuation which is related to the decrease in sound power between two parts in an acoustical system. If the decrease in sound power is desired between two points of a system having equal cross sectional area and the sound may be assumed to be of uniform intensity on each area then the attenuation can be shown to be 20 times the logarithm to the base 10 of the ratio of the sound pressure incident on the system to the sound pressure transmitted.

$$\text{Attenuation} = 20 \log_{10}\left[\frac{\text{P incident}}{\text{P transmitted}}\right] \tag{1}$$

Insertion loss is the difference in dB between two sound pressure levels measured at the same point in space after a muffler is installed between the measuring point and the source.

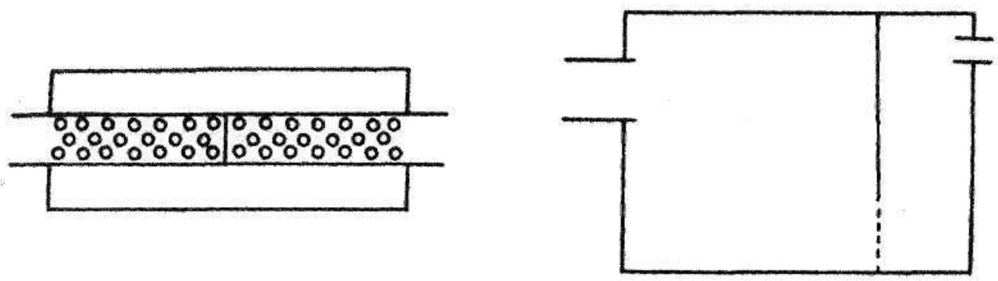
Existing Mufflers

Most small engine mufflers in common use today are neither dissipative or reactive but are instead resistive type elements [32]. When bursts of exhaust gas are forced to pass through holes about 1/4 inch in diameter in resistive elements, the slug of exhaust gas is broken up by the fluid resistance of the holes. Acoustic energy is dissipated as turbulence and heat which is radiated by the muffler shell. A popular element which works on this principle is the single plug two stage dissipative muffler shown in Figure 27a. Theoretical consideration of this type of muffler has not been found in the literature, although Sanders [33] has studied the resistance produced by steady flow through the holes.

A variation of the above design is the expansion pot shown in Figure 27b. It has a chamber into which the slug of exhaust gas may expand before it passes through the holes. The objectives of both of the mufflers is to break up the nonsteady flow so that the exhaust gases leave the system free of large acoustic purturbations. These mufflers cost about $2.

Another muffler that is currently available for model airplanes and is being developed for larger engines is the Murphy muffler. The heart of its operation is a silicon rubber sheath into which exhaust gases expand. Tests [34] on a .61 cubic inch two cycle airplane engine show it to be most effective at frequencies above 500 Hz. In some cases this muffler is said to improve engine performance [34]. Cost is about $15.

The most effective muffler tested in this study was the Hush muffler manufactured by the Hush Company of Ann Arbor, Michigan. It provided 6 dBA insertion loss for the engine with no muffler as compared to a 5 dBA insertion loss



(a) (b)

Figure 27a. Single Plug, Two-Stage Dissipative Muffler

Figure 27b. Expansion Pot

for the stock muffler. Since this muffler was welded together, its theory of operation was not determined. It had low back pressure. Its cost is estimated at more than $20.

Table 5 is a summary of the effectiveness of mufflers now available.

Table 5. Comparison of Existing Mufflers

Muffler	Insertion Loss (dBA)	Back Pressure (in. water)	Cost ($)
Single Plug Two Stage Dissipative	5	6	2
Expansion Pot	5	6	2
Hush	6	3	20

Theories of Muffler Design

There are three theories of exhaust noise of acoustic filters that have been reduced to the point where they can be used for intelligent muffler design. There is a fourth method that is used in industry that is "partly theoretical and largely emperical" [35]. Reference 32 is an excellent summary of the state of the art in automotive muffler design. This method will be avoided here because of the time and experience necessary to obtain successful results with its use. Gatley and Gegesky [36] have proposed the use of an adjustable element as an aid to this method of muffler design.

The first method is the classical acoustical filter theory developed by Stewart [37]. Here an acoustic system is modelled as a combination of basic elements such as fluid resistances, volumes and masses. Analogies can be drawn between these three elements and the spring, mass, damper elements of mechanical systems. The second theory, that of distributed acoustic impedance is quite similar to transmission line theory. It is presented in design form in reference 24. The third, the single pulse theory, has been developed by Davies [38,39] and is based on the theory of shock waves. Each of these theories will be briefly reviewed with careful attention paid to the assumptions made in each. From the three theories, one or a combination will be chosen for use in this study.

Single Pulse Theory

The single pulse theory is based on one-dimensional unsteady flow in pipes and is free of the assumption to be made in the next two theories concerning weak pressure perturbations in the exhaust system. It has the additional advantage that the system back pressure can be predicted. The theory is presented in design form in reference 38.

Included in the development of this theory, however, is the assumption that a shock develops very quickly as a result of the release of gases from the combustion chamber. Davies [39] feels that this theory is not applicable to very short exhaust systems since reflected pressure waves can return soon enough to modify the flow of gases through the exhaust valve.

Acoustic Filter Theory

Development

In 1922, the effects of electric filters suggested to Stewart [37] the possibility of analogous effects in acoustic systems. After making the assumptions listed below he drew a direct analogy between acoustic and electrical systems.

(1) He assumed that the acoustic elements of a system were small relative to the wavelength of sound, allowing him to neglect phase changes within the element. This allowed the second assumption.

(2) An acoustic system is made up of lumped linear

elements such as capacitors and inertances.

(3) Only plane waves are transmitted in the channels and tubes of the system.

(4) The acoustic system is excited by sinusoidal excitation.

Stewart's modelling procedure was based on the three hypothesis listed below.

(1) If an acoustic system is excited by a harmonically varying pressure the resulting volume current in the system is also harmonic.

(2) The product of acoustic impedance and volume current in any portion of the line equals the pressure difference applied.

(3) The sum of the volume currents at a junction is zero.

The analogy between electrical, mechanical and acoustical systems can be drawn as shown in Table 6.

It is also possible to draw analogies between the generators of system excitation. The constant voltage generator of electrical circuitry is analogous to a constant pressure acoustic generator, physically represented by a loudspeaker. A constant current generator is analogous to a constant volume current generator or a pump. The idealized acoustic elements are related to physical parameters by considering the fluid mass, viscosity and compressibility. The relations between the physical parameters and the value

Table 6. Electrical, Mechanical and Acoustical Analogies

Electrical Quantity	Electrical Symbol	Mechanical Quantity	Mechanical Symbol	Acoustical Quantity	Acoustical Symbol
charge	q	displacement	x	volume disp.	x
current	$i = \frac{dq}{dt}$	velocity	$v = \frac{dx}{dt}$	volume current	$\dot{x} = \frac{dx}{dt}$
voltage	v	force	f	pressure	p
inductance	$L = \frac{v}{\ddot{q}}$	mass	$m = \frac{f}{\ddot{x}}$	inertance	$M = \frac{p}{\ddot{x}}$
capacitance	$\frac{1}{C} = \frac{v}{q}$	compliance	$k_s = \frac{f}{x}$	compliance	$\frac{1}{C} = \frac{p}{x}$
resistance	$r = \frac{v}{\dot{q}}$	friction	$r = \frac{f}{\dot{x}}$	resistance	$r = \frac{p}{\dot{x}}$

of the lumped elements will not be derived here, but may be found in any elementary acoustics text [40,41,42]. The term acoustic mass refers to a fluid vibrating without compression in a hole or open-ended tube. It is related to the physical parameters by equation (2).

$$M = \frac{\rho \ell'}{\pi a^2} \tag{2}$$

The length used in equation (2) must be corrected to include the mass of fluid outside of the tube that also vibrates. For air vibrating in a hole, this correction is .82a where a is the radius of the hole. This correction must be applied to both ends, hence ℓ' equals the physical length of the hole plus 1.64a. Acoustic compliance refers to the compression of a fluid in an enclosed volume and is related to the physical parameters of the element as shown in equation (3)

$$C_o = \frac{V}{\rho c^2} \tag{3}$$

The pressure and volume current in each of the elements are related by the relations given in Table 6. Consider an acoustic resistance excited by a harmonically varying pressure source. Using Stewart's first hypothesis, the analysis proceeds as below.

$$r = \frac{p(t)}{\dot{x}e^{jwt}}$$

$$p(t) = r\dot{x}e^{jwt} = pe^{jwt}$$

$$\frac{p}{\dot{x}} = r$$

Similarly for exponential excitation of an acoustic inertance

$$p(t) = M\frac{dx}{dt}$$

$$pe^{jwt} = M\,\frac{d}{dt}(\dot{x}e^{jwt}) = jwM\dot{x}e^{jwt}$$

$$p = jwM\dot{x}$$

$$\frac{p}{\dot{x}} = jwM$$

For an acoustic compliance excited by a volume current

$$\dot{x}e^{jwt} = C\frac{d}{dt}(p(t))$$

$$p(t) = \frac{1}{C}\int\dot{x}e^{jwt}dt$$

$$pe^{jwt} = \frac{1}{jwC}\,xe^{jwt}$$

$$p = \frac{\dot{x}}{jwC}$$

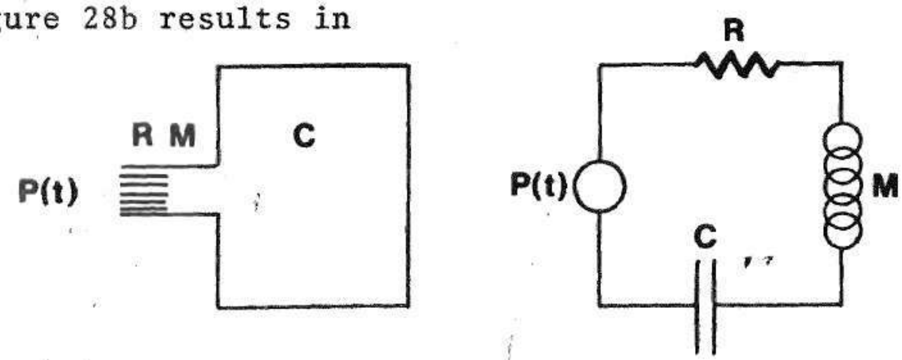
$$\frac{p}{\dot{x}} = \frac{1}{jwC}$$

The complex ratio of acoustic pressure to volume current is defined as the acoustic impedance, Z_a. An acoustic element may be modelled by replacing the element by its acoustic impedance. Acoustic impedances are manipulated in the same manner as electrical impedances, that is impedances in series add algebraically and the recipricals are added for impedances in parallel.

Consider the Helmholtz resonator shown in Figure 28a that is driven by a harmonically varying pressure source having zero internal impedance. The equivalent circuit is shown in Figure 28b. Replacing the elements by their equivalent impedances and adding,

$$Z_{equiv} = r_A + jwM + \frac{1}{jwC}$$

According to Stewart's hypothesis, the product of acoustic impedance times volume velocity in an acoustic circuit is equal to the pressure applied. Applying this to the circuit of Figure 28b results in



(a) (b)

Figure 28. Helmholtz Resonator and Equivalent Circuit

$$\frac{p}{\dot{x}} = Z_{equiv} = r_A + j(wM - \frac{1}{wC}) \qquad (4)$$

The value of the impedance should now be evident, since it is known when the response of the system will be known. Note that the impedance is made up of two parts, a real component called the acoustic resistance and an imaginary component, the reactance. In acoustical radiation problems the sound radiated away from the source is a function of the real part of the impedance.

It would be interesting to consider the electrical system shown in Figure 29a. Assume that the excitation is an ideal voltage source with zero internal impedance. The output voltage will be measured with an instrument having infinite internal impedance. Using an analysis as before, it can be shown that

$$Z = \frac{v}{i} = j(wL - \frac{1}{wC}) \qquad (5)$$

Consider the case where $wL = \frac{1}{wC}$ or when $w^2 = \frac{1}{LC}$. When this

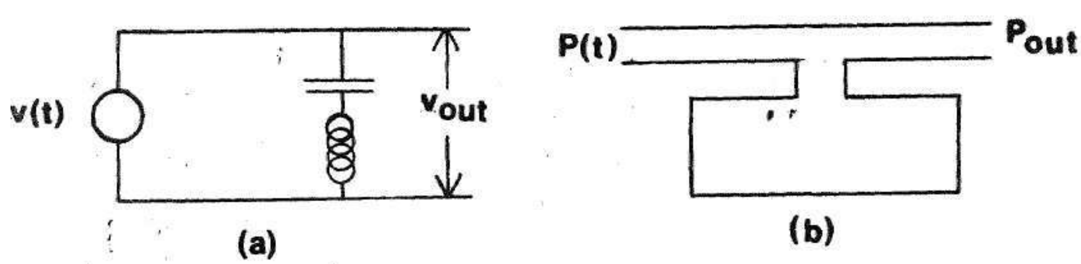


Figure 29. Band Elimination Filters

occurs, Z, the impedance of the branch, becomes zero. Since $v + iZ = 0$ then the output voltage must be zero.

Now consider the analogous system shown in Figure 29b. When $w^2 = \frac{1}{MC}$ the pressure drop at the output is forced to be zero. If the ratio P_{out} to P_{in}, the transmissibility, T, were plotted against frequency it would appear as shown in Figure 30e.

Other acoustic filters are possible using the elements already mentioned and are shown in Figure 30.

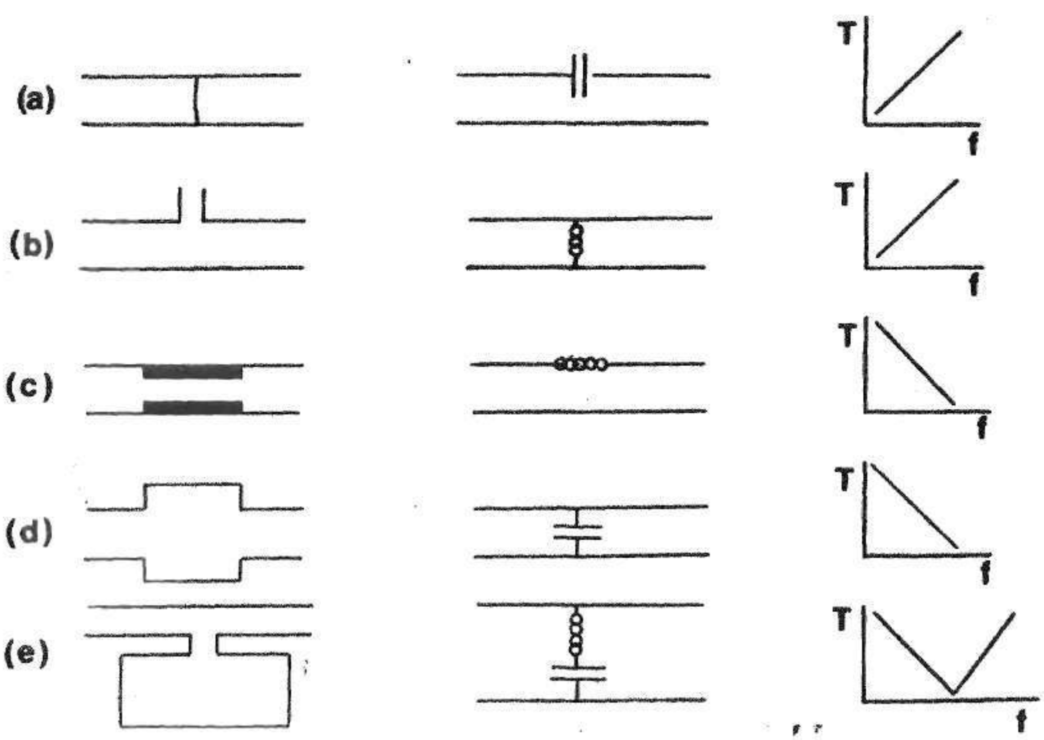


Figure 30. Acoustic Filters

Modeling Muffler Systems

The filters discussed up to this point are simplifications of acoustic systems. To be useful for modelling actual exhaust systems, this theory should be able to include the effects of the source impedance, muffler connecting pipes, and the termination impedance. Most exhaust theories consider the engine a constant volume current source, a seemingly justifiable assumption since no matter what the engine back pressure is, within reason, the volume current remains the same.

Another assumption that is almost always made is that the acoustic impedance of the source is infinite. Sreenath and Munjal [43], consider this an oversimplification and have computed and compared the theoretical attenuation achieved with two system models, one having finite impedance and the other infinite. For the muffler system that they studied they found less than about 3 dB difference in attenuation for frequencies up to 500 Hz. Above 500 Hz errors greater than 20 dB resulted. The point of interest here is that if the exhaust impedance could be found, this method allows its inclusion into the analysis as the impedance of the source is shown in the circuit of Figure 31.

If they can be considered lumped elements, the effects of the connecting pipes can be easily included in the analysis by adding inductances to the circuit in Figure 31.

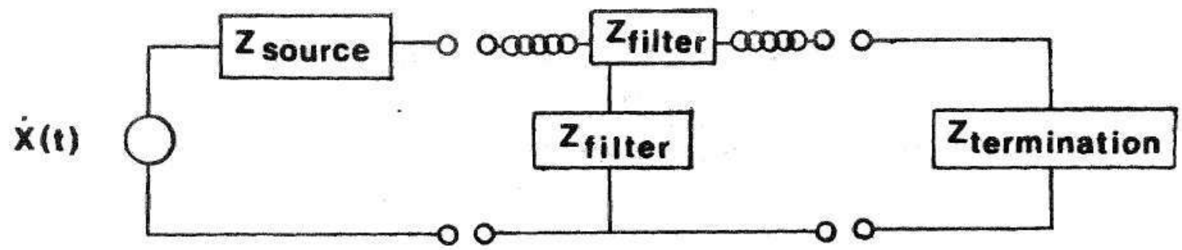


Figure 31. Acoustic Filter Theory Model of Exhaust System

The last important consideration that must be considered in the model is the impedance of the tailpipe termination. A theoretical tailpipe impedance can be found [40] and included in the model as shown. A more in depth discussion of the assumptions of no flow, no thermal gradient and small pressure will follow the discussion of the impedance method.

A shortcoming of this method is in the prediction of the attenuation for an element as shown in Figure 30d. At high frequencies or when the wavelength of interest is of the order of twice the length of the elements, bandpasses will occur that this theory does not predict. This theory also does not consider the dimensions of the filter. It will be shown in the next chapter that the area ratio between the duct and the chamber is important.

To design a muffler system using this theory, an acoustic filter can be chosen from those of Figure 30 or combinations of those which result in acceptable attenuation.

Design charts for resonator type elements can be found in references 42 and 44. These charts assist in selecting resonator dimensions for the required attenuation. Reference 20 gives examples of designs using the acoustic filter method. Once a system has been designed, its performance can often be predicted within 5 dB [20] by using the model of Figure 31.

Distributed Impedance Method

Development

The assumptions made in this theory are identical to those of the filter theory except for the one concerning the phase changes in the system. Briefly, the major assumptions are no flow, no viscous losses, only plane waves, and small acoustic pressures. Consider the system shown in Figure 30d. When the acoustic filter theory is applied to it the transmission predicted will be in error at high frequencies [40]. Obviously, the fluid in the system does not act like an ideal spring since a pressure on one end will produce some volume current on the other end. Phase changes and the effect of area changes in the system are also ignored.

Fundamental to this theory is an understanding of the concept of acoustic impedance. It has been previously defined as the complex ratio of pressure to volume current. The characteristic impedance of a duct of cross sectional area, S_1, is given by

$$Z_{01} = \frac{\rho_o c}{S_1} \tag{6}$$

where $\rho_o c$ is the characteristic resistance of the fluid in the duct and S_1 is the duct cross sectional area.

Equation (7), shown below, is the classic wave equation

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} \tag{7}$$

which describes the propagation of plane waves in a duct. It has the general solution [40,45]

$$p = p_+ e^{jw(t-\frac{x}{c})} + p_- e^{jw(t+\frac{x}{c})} \tag{8}$$

where p_+ and p_- denote waves traveling forward and backward, respectively. By applying equations (6) and (8) and the conditions of continuity of pressure and volume current to an acoustic system such as that shown in Figure 32, it is possible to obtain an expression relating the incident pressure and the transmitted pressure. Note that transmitted pressure refers to that acoustic pressure which is transmitted down the pipe and does not concern the radiated sound. When considering the effects of the tailpipe termination on the sound in the system it is assumed that at the end of the pipe the wave completely reflects with negative

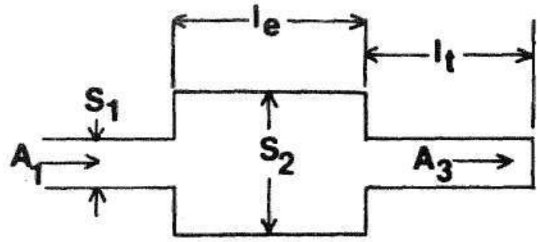


Figure 32. Expansion Chamber

sign, that is a compression wave reflects as a rarefaction. This assumption is based on the results of a study by Levine and Schwinger [46] who theoretically determined the reflection factor for an unflanged pipe at low frequencies. The radiation of sound from the pipe will be discussed in the next section.

Expressions relating the incident and transmitted pressure A_1 and A_3 for a number of systems are shown below (see Appendix G). Note the parameters of interest for each system. The cutoff frequency is that frequency below which no attenuation is predicted.

<u>Expansion Chamber with Infinite Tailpipe</u>

Parameters: ℓ_e, $m = \frac{S_2}{S_1}$

Attenuation = $10 \log_{10} \left|\frac{A_1}{A_3}\right|^2$

where

$$\left|\frac{A_1}{A_3}\right|^2 = 1 + \frac{1}{4}(m-\frac{1}{m})^2 \sin^2 k\ell_e \qquad (9)$$

<u>Expansion Chamber with Finite Tailpipe</u>

Parameters: ℓ_e, ℓ_t, m

$$\text{Attenuation} = 10 \log_{10} \left|\frac{A_1}{A_3}\right|^2$$

where

$$\left|\frac{A_1}{A_3}\right|^2 = 1 + \frac{(m^2-1)^2}{2m^2}\sin^2 k\ell_e - \frac{m^2-1}{2m}\sin^2 k\ell_t \sin^2 k\ell_e -$$

$$\frac{m^4-1}{2m^2}\cos k\ell_t \sin^2 k\ell_e \qquad (10)$$

$$\text{cutoff frequency, } f_c = \sqrt{\frac{4+\frac{2\ell_e}{m\ell_t}}{(m+\frac{1}{m})\ell_e\ell_t}} \qquad (11)$$

<u>Single Chamber Resonator with Finite Pipe</u>

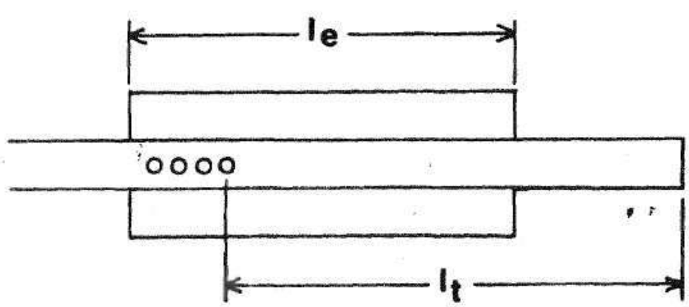


Figure 33. Single Chamber Concentric Resonator

Parameters: c_o, V, ℓ_t, f_r

Attenuation =

$$10\log_{10}\left[1+\frac{\sqrt{\frac{c_oV}{S}}\,\sin 2k_r\ell_t\,\frac{f}{f_r}}{\left(\frac{f}{f_r}-\frac{f_r}{f}\right)}+\frac{\frac{c_oV}{S^2}\sin^2 k_r\ell_t\,\frac{f}{f_r}}{\left(\frac{f}{f_r}-\frac{f_r}{f}\right)^2}\right] \tag{12}$$

$$f_c=\frac{f_r}{\sqrt{1+\frac{c_o\ell_t}{2S}}} \tag{13}$$

Modeling Muffler System

Sreenath and Munjal [43] have applied the previous theory to model muffler systems and have extended it to include the effects of the source or tailpipe. As already mentioned, they found less than 3 dB theoretical error with the assumptions of a completely reflecting tailpipe for frequencies up to 500 Hz and less than 5 dB theoretical error up to 500 Hz with the assumptions of a nonreflecting source as in the previous analysis. Davis [47] also found less than 0.1 dB error when the tailpipe was assumed to be totally reflecting. Although the acoustic filter theory has the advantage that the source and termination impedance may be easily included, its inclusion does not seem necessary below 500 Hz. Davis et al. [47] have plotted the attenuation equations describing expansion chambers and resonators and present a method whereby muffler systems may be designed using this method.

Evaluation of System Models

Theories have been presented which allow the generation of a model of an engine exhaust system. If the assumptions made in the construction of the model are justifiable when compared to the actual system, then the model should provide an acceptable means of predicting the performance of or for designing mufflers. The following assumptions were implicit in the system models.

(1) The effect of flow in the system is negligible.

(2) The effect of a temperature gradient is negligible.

(3) Sound pressures are assumed small allowing the neglection of nonlinear effects.

(4) Only plane waves are propagated in the system.

To objectively evaluate the model that the two theories allow, the assumptions of the analysis must be evaluated in terms of acoustic considerations such as plane waves, sound intensities, and system geometry.

Plane Waves

When the half wavelength of interest becomes less than the maximum diameter in the system, then the assumption of plane waves will be violated [40,47]. At 1000°F, c, the speed of sound in air, is equal to 1880 feet per second. Since the wavelength, λ, equals $\frac{c}{f}$, then 1/2 λ at 500 Hz equals about 1-1/2 feet. If the system diameter is 1 foot, there should always be plane waves in the system at frequencies below 500 Hz.

Acoustic Pressure Intensities

By assuming that the acoustic pressures in the system are small it is possible to consider the system elements linear, that is, nonlinear effects in the resistors and impedance can be ignored. Beranek [42] considers nonlinear effects important after about 120 dB. Although it is invalid here, this assumption greatly simplifies the analysis, that is, it makes a theoretical analysis feasible and will be justified for that reason.

Flow

Watters [20] et al. show both experimentally and theoretically how flow through the system increases the acoustic resistance, thereby increasing the attenuation obtainable in a system. Davis [47] feels that since the flow velocities in the system are generally less than Mach .1, then the effects of flow on the wave motion in the system will be negligible.

Temperature Gradient

Although temperature gradients may exist throughout the system, Davis [24] feels they have little effect on the filter performance. This is partly justified when equation (3) is examined. Remembering that c^2 is proportional to T, and ρ is proportional to T^{-1} it becomes evident that the chamber impedance is not a function of temperature.

Experimental Verification of Theories

In spite of the above assumptions, both the filter theory and the impedance theory can predict results with less than 5 dB error at frequencies up to 500 Hz. Fukuda [49], Davis [47] and Watters [20], all show how the theoretical predictions of the two methods agree with insertion losses or attenuation resulting on actual engine applications.

Design of Muffler Systems

Attenuation

Figure 7 shows that the major component of exhaust noise is in the range 25-300 Hz. To reduce the A-weighted level, the attenuation should be the greatest at the highest frequency, 300 Hz, and could be somewhat less at lower frequencies. From an annoyance standpoint the A-weighting means little when considering the low frequency tones. The best muffler should reduce the intensity of all of the low frequency tones and have a cutoff frequency as low as possible.

At 300 Hz the difference between the exhaust noise and the system noise is 1-1/2 dB. Subtracting the radiation and blade noise from the noise of the complete system as shown in Figure 7 reveals that the radiation and blade noise at 300 Hz is 74-1/2 dB and the exhaust noise is 74-1/2 dB. To reduce the exhaust noise at 300 Hz to the point where it does not contribute to the total noise requires that it be quieted to 10 dB less than the remainder of the system or to 64-1/2 dB. A design attenuation of 10 dB is therefore chosen with a desired attenuation as shown in Figure 34.

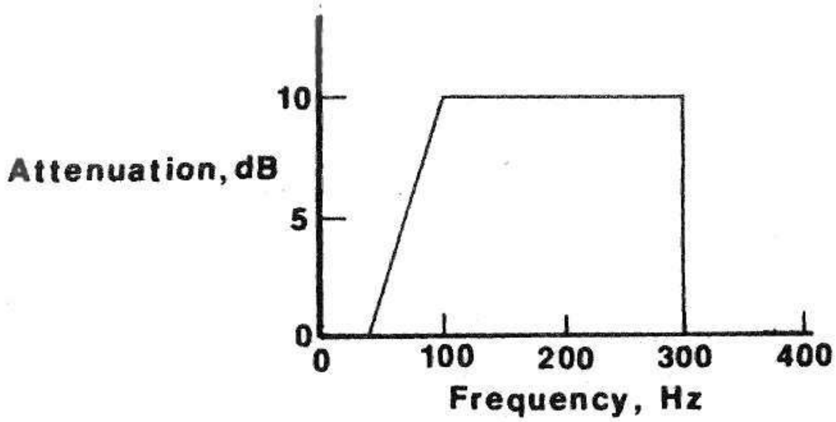


Figure 34. Design Exhaust Attenuation

Summary of Constraints

From the previous discussion the following constraints on the muffler design may be listed.

(1) The system should be as small as possible.

(2) The system should provide the attenuation shown in Figure 34.

(3) The lower cutoff frequency should be as low as possible, preferably 40 Hz.

(4) The flow velocities in the system should not be greater than 10,000 feet per minute [32].

(5) The exhaust system back pressure should be less than 10 inches of water [20].

(6) The exhaust system should be low in cost and easy to manufacture.

(7) The system must be durable.

Design

Since it can satisfactorily model any system that the acoustic filter theory can and since the design method in the impedance theory is more advanced, the impedance method was chosen to design the muffler system.

The following systems have been presented in design form in reference 47: resonators, volume changes, and multiple volume changes. Each of these systems will be considered in order to determine the optimum system which will be the smallest one that satisfies the attenuation requirement.

The area of the exhaust pipe must be known before any of the designs can be made. The average temperature in the exhaust pipe was measured to be about 1000°F. If there was no blow by and the air in the cylinder was completely removed after each power stroke, then the volume of air into the engine would be

$$\text{displacement x rpm x } \frac{1}{2} = 10 \text{ x } 3000 \text{ x } \frac{1}{2} = 15000 \ \frac{\text{in}^3}{\text{min}}$$

Since the air came in at 70°F and went out at 1000°F, the volume out would be

$$\text{volume in x } \frac{\text{Tout}}{\text{Tin}} = 15000 \cdot \frac{1460}{535} = \frac{40{,}000 \text{ in}^3}{\text{min}}$$

The maximum flow in the system should be 10,000 feet per minute, hence the pipe area should be

$$S = \frac{40,000}{120,000} = .333 \text{ in}^2$$

or the pipe radius should be .326 inches. The existing pipe has an inside radius of .311 inches and is considered satisfactory.

As an aid to the design of muffler system, Davis et al. have plotted the attenuation predicted for various systems for a number of parameters. These plots allow the selection of system dimensions such that the desired attenuation will result. This is the first step of the design. The second step is to compute the necessary tailpipe length to get the required low frequency cutoff.

The last step is to check the passbands of the tailpipe. Tailpipe passbands will occur at frequencies whose half wavelengths and multiples thereof correspond to integer multiples of the tailpipe length. They will be predicted by equation (14). At the passband frequency little and

$$f_{passband} = \frac{nc}{2\ell_t} \tag{14}$$

sometimes negative attenuation may result [47]. This is not due to exceptions in the first law of thermodynamics, but is a result of the tailpipe providing more efficient acoustic coupling between the engine and the atmosphere and surroundings than the engine with no muffler. If the passband occurs in the range where the attenuation is desired, the muffler will be ineffective and the system must be redesigned. An example will illustrate the design method.

<u>Single Expansion Chamber</u>. The system is shown in Figure 35. The parameters to be established are m, l_e, and l_t.

Figure 36 is a design chart which is taken from reference 47. It is a plot of equation (9). Ten dB attenuation is desired between 100 and 300 Hz. At m = 9 there will be 10 dB attenuation between $kl_e = .8$ and $kl_e = 2.4$. Note that no recommended procedure for choosing m and kl_e is given with the charts other than to have symmetrical attenuation on the curves and that was the reason m and kl_e were so chosen.

Since $kl_e = .8$ and $k = \frac{2\pi f}{c}$ at 100 Hz

$$l_e = \frac{.8c}{2\pi f} = \frac{.8(1580)}{2\pi(100)} = 2.39 \text{ feet}$$

From equation 11

$$f_c = \frac{c}{2\pi}\sqrt{\frac{4+\frac{2l_e}{ml_t}}{(m+\frac{1}{m})l_e l_t}}. \tag{11}$$

Since m, f_c, and l_e are known (11) can be solved for l_t when m = 9, l_e = 2.39 and f_c = 40 l_t = 10.3 ft. The complete system length is 10.3 + 2.39 = 12.69 feet, obviously too large.

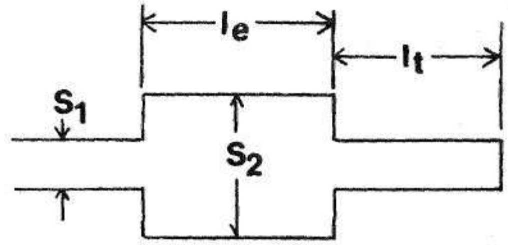


Figure 35. Single Expansion Chamber with Finite Tailpipe

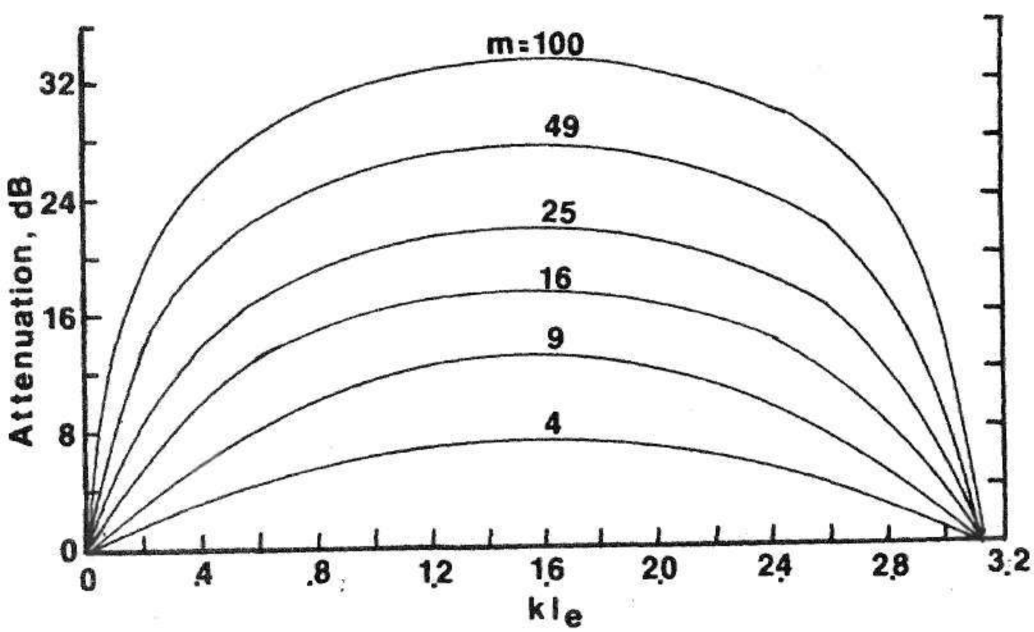


Figure 36. Single Expansion Chamber Design Curves

A tailpipe passband will also occur according to (14) at

$$f = \frac{nc}{2l_t} = \frac{1880}{2(10.3)} = 94 \text{ Hz}$$

If a number of systems are designed, varying m and kl_e, it soon becomes apparent that attainment of the required cutoff frequency is the constraint that requires long systems. Examination of equation (11) reveals that if the product of m and kl_e were as large as possible then l_t could be shorter and still get the required cutoff frequency. A number of systems were designed for various values of the parameter ml_e and the results are shown in Table 7.

Table 7. Expansion Chamber Design

ml_e (feet)	l_e (feet)	l_t (feet)	m	$l_e + l_t$ (feet)	$f_{passband}$ (Hz)
22.3	.45	10.	49	10.4	94
22.3	2.39	10.3	9	12.6	91
45	2.8	5.	16	7.8	188
147	3.	1.53	49	4.53	614

The conclusions are that no system of this type seems acceptable because of the size. To optimize the length of this system choose ml_e as large as possible.

Multiple Expansion Chambers. The effect of multiple expansion chambers is to increase the attenuation possible [47]. If the dimensions are chosen properly then some element passband frequencies can be eliminated [47]. The two chamber system will not help reduce the cutoff frequency. Multiple expansion chambers appear to have no advantage when the optimum size system is desired.

Single Chamber Concentric Resonator. The single chamber concentric resonator is shown in Figure 37. The parameters to be established are the volume, the element and tailpipe lengths, and the number of holes and their diameter. The hole dimensions may be combined into a single parameter, c_o, which is a function of the number of holes, their size, and the thickness of the pipe in which the holes are drilled. If the chamber dimensions are less than about $1/8\lambda$ the attenuation for this system can be predicted by equation (12).

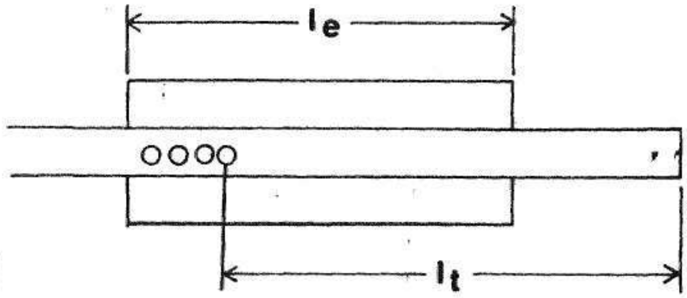


Figure 37. Single Chamber Concentric Resonator

Note that the attenuation

$$\text{Attenuation} = 10\ \log_{10}[1+\sqrt{\frac{c_oV}{S}}\ \frac{\sin 2k_r l_r \frac{f}{f_r}}{(\frac{f}{f_r} - \frac{f_r}{f})} + \frac{c_oV}{S}\ \frac{\sin^2 k_r l_r \frac{f}{f_r}}{(\frac{f}{f_r} - \frac{f_r}{f})}] \quad (12)$$

$$f_c = f_r / \sqrt{1+\frac{c_o l_t}{2S}} \quad (13)$$

is a function of the parameters $\frac{f}{f_r}$, $k_r l_t$, $\sqrt{\frac{c_oV}{2S}}$; and also note that as l_t increases the cutoff frequency decreases.

Using the same design steps as in the single expansion chamber design, only with design charts prepared from equation (12), the dimensions of a number of systems satisfying the attenuation requirements were calculated. All of the systems designed were too large and there was no obvious trend which would lead to a smaller system.

A computer program was written to try to find the optimum system dimensions. The method used was an exhaustive search technique in which the resonant frequency, the volume, and the length were varied over the entire range of interest as the shortest, or smallest volume system satisfying the attenuation requirements was searched. A printout of the computer program is shown in Appendix H. The program converged to a true optimum only when the smallest volume was searched. When the shortest length was searched, the program converged to the longest diameter and vise versa.

The computer program considered about 9000 different muffler systems with parameters varying as follows: resonant frequency, 40-100 Hz; volume, .1-.4 cubic feet; length, .1-.8 feet. The smallest system considered which satisfied the attenuation requirements had a resonant frequency of 60 Hz, a volume of .25 cubic feet, a diameter of .65 feet, and was .76 feet long. Note that this system is not the shortest one, but is the one with the smallest volume. It is too large to be acceptable.

Summary

Within the assumptions made in this analysis, namely no flow, complete reflection at the tailpipe termination, no viscous losses and small acoustic pressures, no acceptable solutions were found when expansion chamber and single chamber resonator type systems were investigated. A method of optimizing the length of expansion chamber systems was found.

Muffler Termination

Introduction

The previous designs were a failure for two reasons. First, the tailpipe had to be too long to get satisfactory low frequency attenuation. Second, the tailpipe had resonant frequencies at which there was little attenuation. When the tailpipe became short enough to meet the size constraints it had a resonant frequency well up in the audible range.

The resulting system could be worse than no muffler at all [47].

The previous conclusions would probably come as no surprise to the manufacturers of small engine mufflers. The study helps to explain why emperically derived mufflers like those of Figure 27 have been developed instead of more conventional resonators or expansion chambers.

An intelligent design would examine the failure of a certain design and the assumptions in the design model that might be in error. The three areas of particular interest include the failure at low frequencies, the effects of the tailpipe, and the complete reflection assumption. These three areas imply that a more effective muffler system might be one that could obtain low frequency attenuation without a tailpipe. Radiation from the muffler should also be considered in the model.

Acoustic Radiation from a Hole

Consider a pressure wave A_1 incident on the open end of a pipe as shown in Figure 38. At the end of the pipe a wave B_1 will be reflected and a wave A_2 will be transmitted. Define the reflection coefficient, α_r, to be equal to $|\frac{B_1}{A_1}|^2$, the transmission coefficient, α_t, to be $1 - \alpha_r$, and the reflection factor, R, to be $\frac{B_1}{A_1}$. Another,parameter that will be useful here is ka, the product of the wave number, $k = \frac{2\pi f}{c}$ and the pipe radius a.

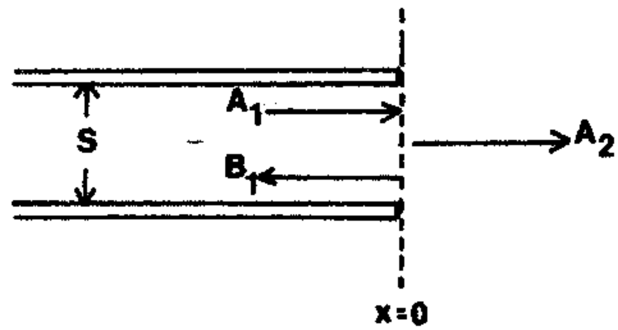


Figure 38. The End of a Hole

Using the same technique as in the last section it can be shown that

$$\frac{B_1}{A_1} = \frac{Z_o - \frac{\rho_o c}{S}}{Z_o + \frac{\rho_o c}{S}} \tag{15}$$

where Z_o is the acoustic impedance of the termination. To find $\frac{B_1}{A_1}$ at x = 0 requires that Z_o be known. By considering the similar case of a piston in the end of a pipe it can be shown [40] that

$$Z_o = \frac{\rho_o c}{S}\left(\frac{k^2 a^2}{4} + j.6ka\right) \tag{16}$$

Substituting (16) into (15) and neglecting second order terms of ka

$$\frac{B_1}{A_1} \cong -1 + j1.2ka \tag{17}$$

$$\alpha_t = 1-\alpha_r \cong (ka)^2 \tag{18}$$

Levine and Schwinger [46] have rigorously shown that as ka becomes very small the reflection factor at the end of an unflanged pipe is

$$\frac{B_1}{A_1} = -e^{-1.23jka} \tag{19}$$

$$\frac{B_1}{A_1} = -\cos(1.23ka)+j\sin(1.23ka) \tag{20}$$

$$\cong -1 + j1.23ka \tag{21}$$

Since the reflection factor predicted by the piston approximation is quite close to the exact value, it is assumed that the transmission coefficient predicted in (18) is also close to the true value. Levine and Schwinger also note that the radiation into the pipe is quite similar to radiation out of the pipe for plane waves normal to the hole.

The attenuation provided by the unflanged pipe termination is given by

$$\text{Attenuation} = 10 \log_{10}\left(\frac{1}{k^2a^2}\right) \tag{22}$$

Using a similar method to that above and considering the radiation from an infinite baffle, it can be shown that the attenuation of a flanged pipe is

$$\text{Attenuation} = 10 \log_{10}\left(\frac{1}{2k^2a^2}\right) \qquad (23)$$

Comparing (22) and (23) it becomes evident that the attenuation provided by the flanged pipe is 3.0 dB less than that of the unflanged pipe.

The seemingly bold assumption concerning the complete reflection of the sound at the end of the tailpipe can now be investigated. With a tailpipe of .3 inch radius and for frequencies between 50 and 300 Hz at 1000°F, ka ranges from .004 to .025. The reflection factor will be the smallest at the larger values of ka. From the real part of (20), R at 300 Hz equals .999528. R can also be approximated from equation (24).

$$R \cong 1 - (ka)^2 \qquad (24)$$

For the above situation, equation (24) predicts R = .99968. The attenuation can be predicted from (22). Figure 39 shows the theoretical attenuation and reflection factor at the end of an unflanged circular pipe.

Figure 39 shows how the pipe may be considered to be totally reflecting even though much sound gets out. Within the major assumptions of small pressures, and no flow, the results show that the attenuation provided by the unflanged

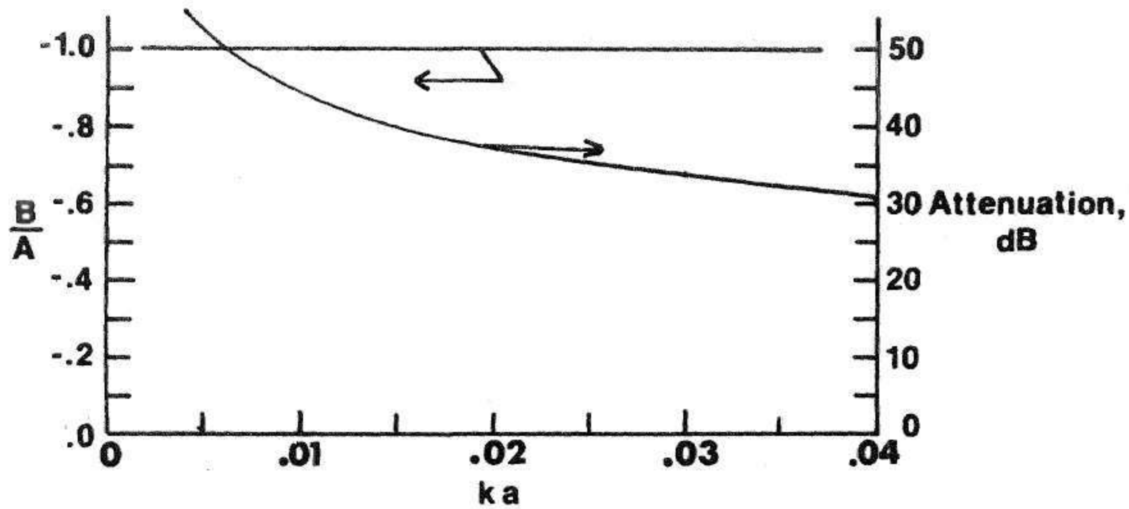


Figure 39. Attenuation and Reflection Factor for Unflanged Pipe

pipe increases as frequency decreases or as the pipe radius decreases.

Application of Theory

Figure 39 suggests that an improved muffler would result if the cross sectional area of the tailpipe termination were very small. The requirements for low back pressure requires that if the cross section of the tailpipe or hole size becomes very small then a number of them must be used to insure adequate flow through the system.

Predicting the effect of multiple holes is complicated by the fact that the sound emerging from various holes is neither totally coherent nor is it totally incoherent. The two extreme cases are considered in Appendix I in the following manner. With one hole the attenuation will simply be that due to one hole, that is if one hole attenuated 20 dB at 200 Hz and the sound inside the pipe was 150 dB, then the sound just outside the hole would be 130 dB. With two holes, each would radiate 130 dB. If the sounds from the two holes were totally coherent their total would be 136 dB, but if their sounds were totally incoherent then the total radiated sound would be 133 dB.

For a given maximum flow rate through the holes, the number of holes, their size, and the resultant attenuation are all related. Considering both of the above cases, it is found in Appendix I that the incoherent method leads to an optimum situation having small holes and the coherent method leads to an optimum of one big hole.

Applying the previous discussion, a muffler was made. It is a half inch pipe with a closed end having a number of holes drilled in the tube wall. The hole size was chosen to be as small as possible yet large enough to insure that they would not become clogged with carbon deposits. The number of holes was chosen such that the bulk flow velocity through the holes would be about 10,000 feet per minute.

Sound pressure level measurements were made inside the pipe using the microphone probe. No SPL variation in the radial direction was observed. The axial variation in SPL was as shown in Figure 40 which implies that the most efficient muffler should have the holes away from the capped end.

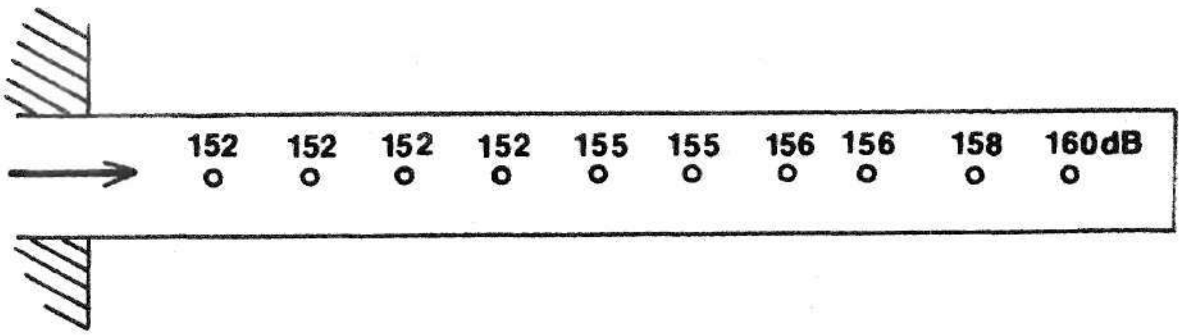


Figure 40. Axial Variation of SPL in Pipe Muffler

Results

Curve A of Figure 41 shows the attenuation predicted for a system with a single hole whose diameter is 0.09375 inches when only one end of the hole is considered. If the effects of both ends of the hole are considered the attenuation should be about twice as much and is so shown in curve B of Figure 41.

Curve A of Figure 42 shows the frequency content of the sound inside of the pipe. Curve B is the frequency content of the noise 2 inches outside of the pipe with the microphone placed in the jet of one hole, hence the effects of multiple holes should not influence the results.

Figure 42 shows that there is a considerable amount of high frequency noise present in the noise emitted by the pipe. Some of this could be radiation from the engine or it could be jet noise. To attenuate the high frequency noise and direct the noise away from the operator, the system shown in Figure 43 was built and placed over the pipe muffler.

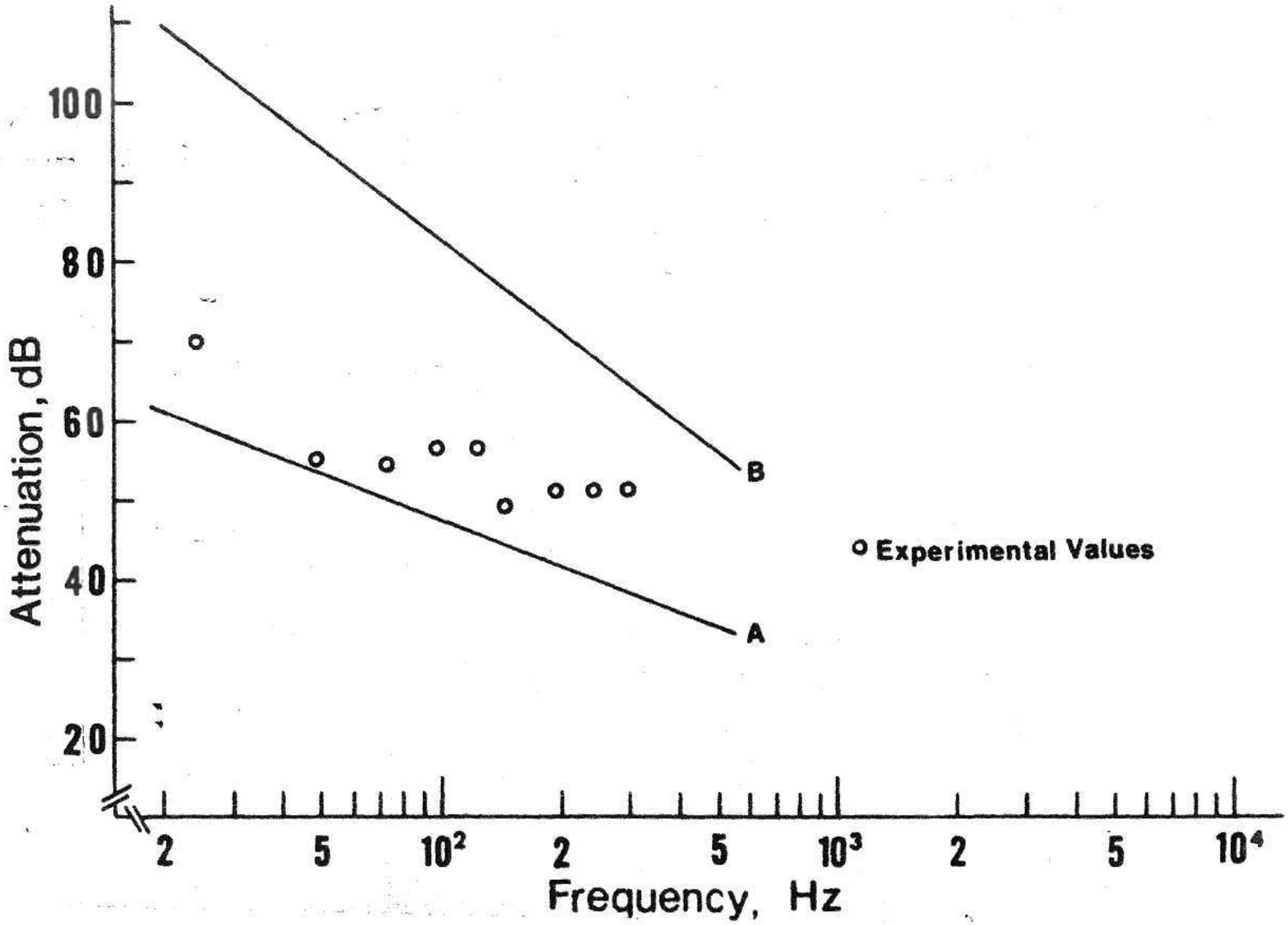


Figure 41. Pipe Muffler Attenuation

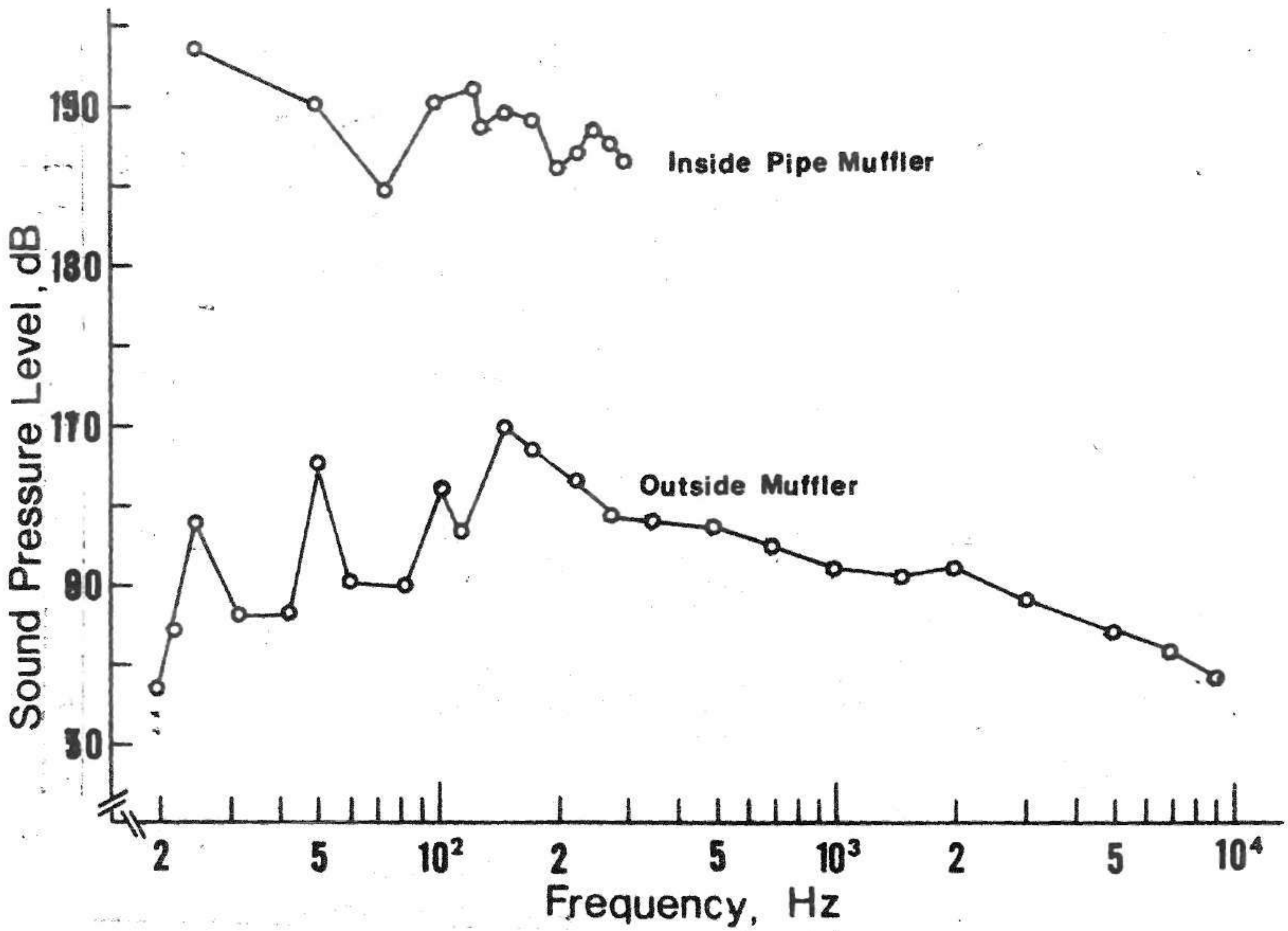


Figure 42. Pipe Muffler SPL

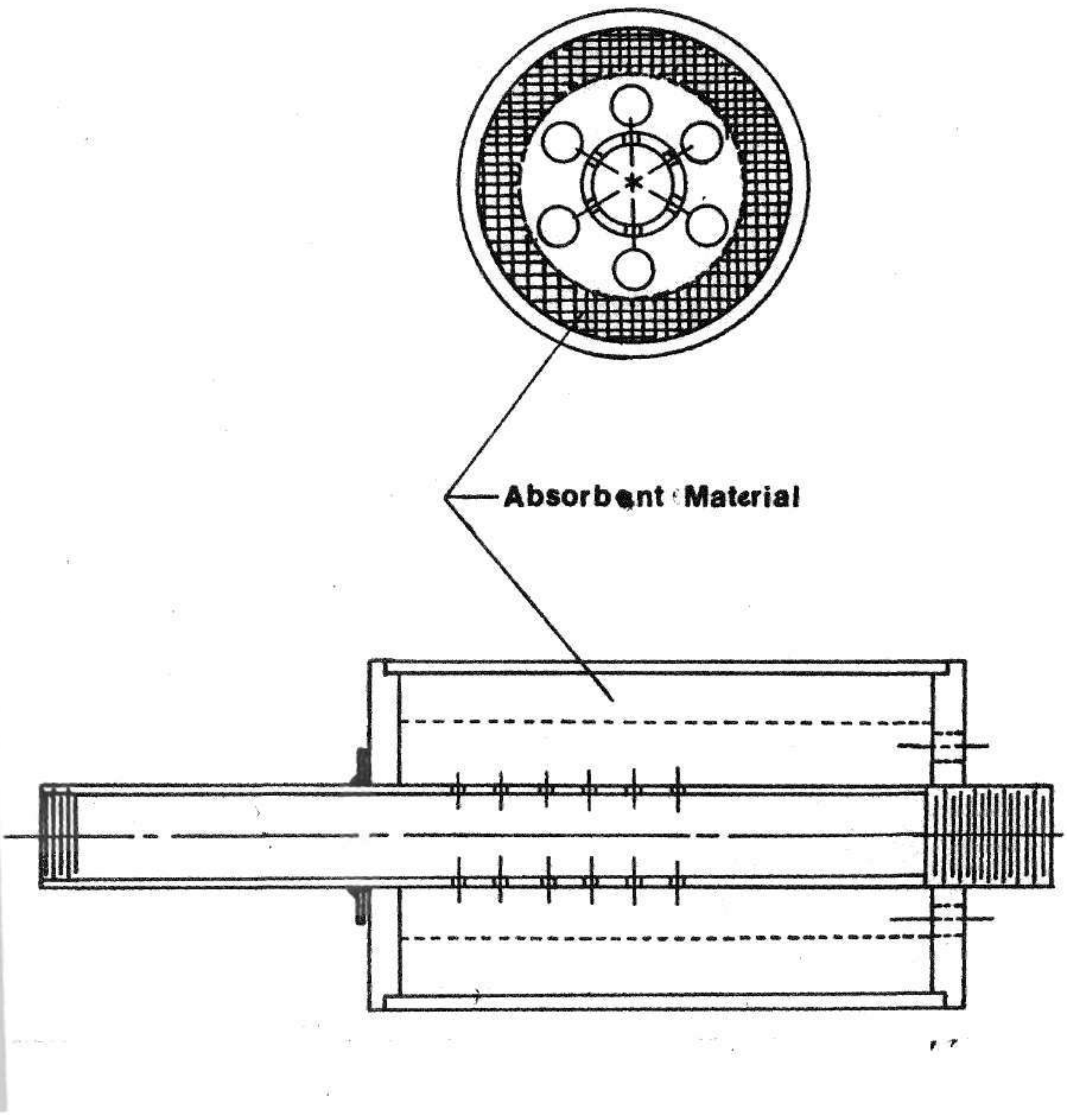


Figure 43. Pipe Muffler

Employing this system the attenuation was improved by 2-1/2 dB when the mower was operated on grass. The pipe muffler with the dissipative element had 4 inches of water of back pressure.

Conclusions and Recommendations

The A-weighted sound levels changed little when the mufflers were changed for the condition where the mower was operating on asphalt. The sound level was 88 dBA for the stock muffler, the pipe muffler and the pipe muffler with the dissipative element. When the mower was operated on grass, changes in A-weighted level shown in Table 8 resulted when various mufflers were used.

Table 8. Mower Noise Levels with Pipe Muffler

Mower operating in grass with blade 2 and engine isolator	
Mode	dBA
No muffler	92
Stock muffler	88½
Pipe muffler	88½
Pipe muffler with dissipative element	86

The reason there was no change in levels on asphalt becomes evident when Figure 9 is examined. When the mower is operated on absorptive grass, the blade and radiation noise is absorbed making the high frequency exhaust noise dominant. On asphalt, the radiation and blade noise is dominant and reduction of exhaust noise has little effect.

The effects of flow have not been considered in this analysis. Davies and Alfredson [48] have studied the effect of flow on the radiation of sound from large exhaust pipes. They present a method that may be used to predict the increase in radiated sound when flow is present. It is questionable whether their results are directly applicable to this situation, since the flow velocity in long tailpipes is steadier than the intermittent flow through the holes of the pipe muffler.

Another effect that was not considered is the nonlinear effect resulting from the high sound pressure levels of the system. Ingard and Ising [50] show that the acoustic linearities that were tacitly assumed when finding the radiation from the holes are not true at sound pressure levels above 130 dB.

Although the effects of flow and high acoustic pressures were neglected, the results of Figure 41 show fair agreement with the theory. The effectiveness of this type of silencer could no doubt be improved if the effects of flow, high acoustic pressures and multiple holes were considered in the analysis and design.

CHAPTER VII

CONCLUSIONS

At its current level of about 90 dBA at the operator, lawn mower noise does not pose a hearing loss damage risk for typical exposure times of one hour. Although projected lawn mower noise recommendations of 85 dBA at the operator and 65 dBA at 50 feet seem conservative, attainment of this goal should result in negligible annoyance resulting from lawn mower noise.

Previously reported data concerning lawn mower noise gave little mention of the surface upon which the mower was operating. Tests in this study show that overall levels can change by as much as 3 dB when the surface is changed.

It was found that in addition to changes in surface, mower noise is a function of load and rpm. The noise from the mower does not change appreciably when grass is actually being mowed.

Mower noise can be broken down into the areas shown in Figure 44. With respect to reducing the A-weighted level, the most important noise is that resulting from mower vibrations. Balancing the blade can result in a 1 dBA reduction in the system noise. On the mower tested, noise radiated from the blade enclosure was dominant over exhaust

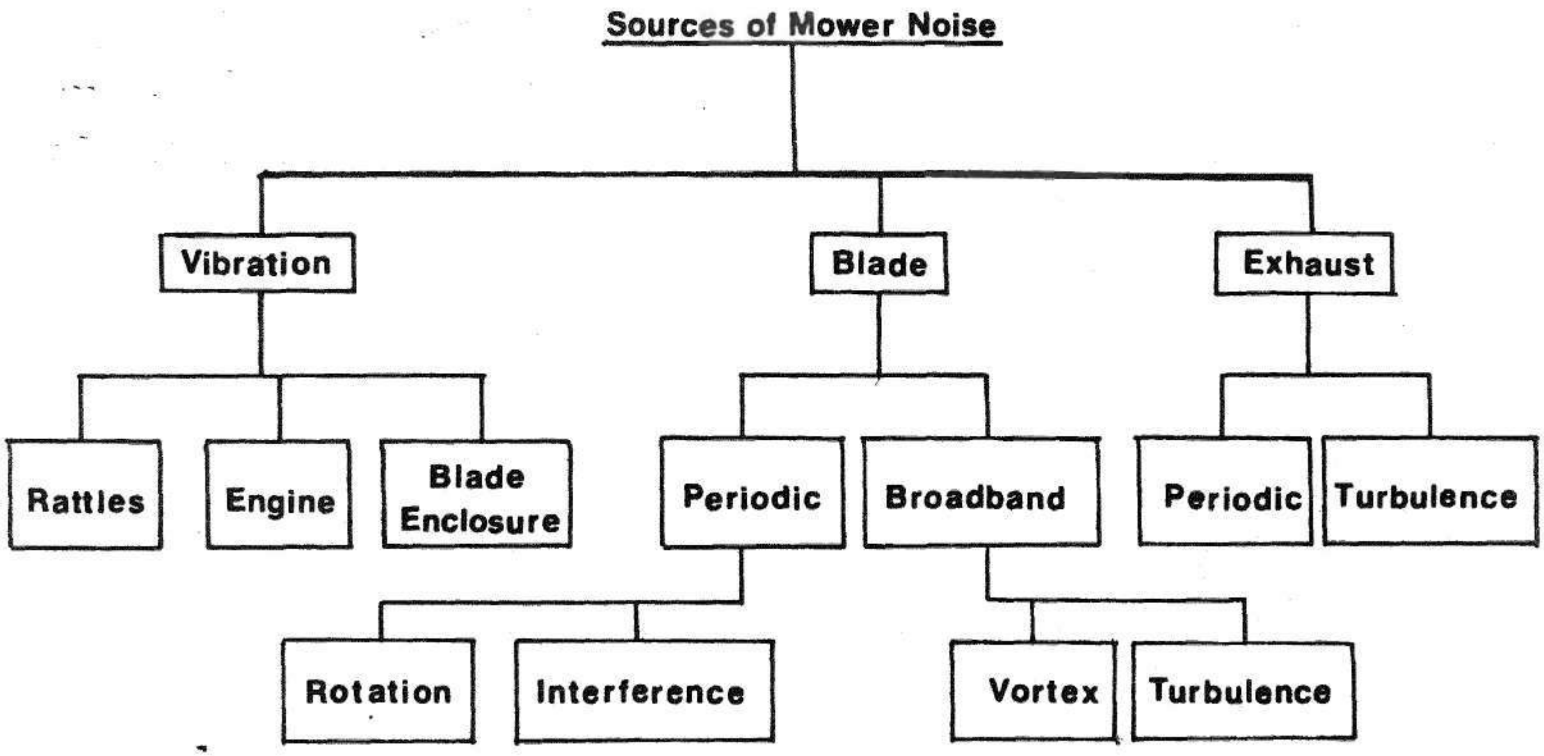


Figure 44. Sources of Mower Noise

and blade noise. Isolating the engine from the blade enclosure resulted in about a 2 dBA reduction in the system noise.

After vibration induced noise is reduced and exhaust noise becomes dominant, a 2 dBA reduction of system noise is possible with the use of a better muffler. Blade noise is the next dominant source. Changing the blade lift and sharpening the trailing edge has no effect on blade noise. Table 9 is a summary of the noise abatement procedures developed that can be effective in reducing mower noise.

Table 9. Mower Noise Solutions

Method	Noise Reduction Possible dBA
Balance Blade	1
Soft Isolator Between Engine and Blade Enclosure	2
Improved Muffler without Isolator	0
Improved Muffler with Isolator	2
Total	5

CHAPTER VIII

RECOMMENDATIONS

Experimental

Since the three noise sources produce noise of much the same quality, it would be advantageous to devise better methods of separating noise sources, particularly the vibration induced noise from the system noise. This would help in identifying sources of vibration noise. Some possible methods might include motoring the engine with an electric motor or shaking the shroud to find its resonant frequencies and modes.

Further research of this type should not be attempted without a graphic level recorder to record the data. The best method of taking data would be with a real time analyzer, especially if the noise of cutting grass is to be analyzed.

Since the noise of the mower is a function of the surface on which the mower operates, it is recommended that future research be conducted on a standard surface. The SAE recommendations for measuring mower noise could be improved from specifying a "surface which is typical of the particular machine application" to specifying concrete or some other standard surface.

Noise Control

Since much of the mower noise is due to vibration, the use of a Wankel or other rotary type engine seems advantageous. A dynamically balanced conventional engine would also reduce mower noise by reducing the amount and intensity of vibration induced noise.

The exhaust muffler developed in this project could be investigated for improvement in the following areas: first, the type and thickness of absorbent material; second, the system dimensions; and third, the number and size of holes in the pipe.

After vibration and exhaust noise have been reduced, the blade noise becomes dominant. Since blade noise increases with rpm, it would be advantageous to operate the mower as slowly as possible and an effort should be made to find what the slowest speed is. Further research on lawn mower noise should concentrate on the blade noise, specifically the effects of the shroud. Parameters that might be investigated include the tip clearance between the blade and shroud and the clearance between the blade surface and the top of the shroud.

Although they were initially dismissed as insignificant, the noise from the miscellaneous rattles becomes significant when the other sources are reduced. The pull rope mechanism, the wheels, and the gas tank were all obvious sources of noise in the improved mower. Any serious design of a quiet mower should make an effort to reduce noise in this area.

With the isolator and improved muffler, the mower powered by the internal combustion engine was quieter than a comparable electric mower, the electric being 89-1/2 dBA on asphalt. This is due to the fact that the electric mower operates at 4000 rpm while the other operates at 3000 rpm. The electric mower would certainly be improved were it designed to run at a lower speed.

APPENDIX A

ACCEPTABLE COST OF NOISE CONTROL

The following is an attempt to arrive at a number which would relate the decrease in noise level to the increase in cost that typical customers are willing to bear. The best that could be hoped for here is a very crude approximation since a more thorough study could include the person's attitude toward the noise, over what range the noise reduction occurs, is the noise tonal in quality, and does the customer associate power with the noise.

What value scale for noise could be used, change in dB, dBA, noys, phons, or rather some noise rating method that considers exposure time? Also, would the value of less noise be related to money or some other value scale? Further, how does one gather data to arrive at conclusions in this area? An opinion poll has obvious drawbacks, among them the fact that most people do not understand dB's, noys or other noise measures.

The method attempted here was to gather information on noisy products that are now on the market and are offered with a special quiet option at some additional cost. Hopefully, the decrease in noise could be plotted against the increase in cost for a number of products. If it were known which of the quieter products were an economic success due to voluntary consumer purchases, i.e.: the customer is not bound by law to buy the quieter product, then an upper limit as to cost per reduction in noise could be found.

A question arises concerning whether the results should be plotted as percent increase in cost or just increase in cost. Although it could be argued that percent increase in cost would be poor logic, nonetheless it is probably the way people think. For example, compare a 10 dBA reduction in noise levels of a $13,000 garbage truck and a $50 lawn mower at additional costs of $125 and $25, respectively. Although the noise control on the garbage truck costs 5 times that of the lawn mower, the customer would more easily accept the higher costing noise control since the percent increase in cost for the product is less. Since customers seem to think in terms of percent increase in cost, the results are so presented.

The question of a noise value scale was answered in the data since most responses were either, "the new machine is ___ dBA quieter" or was "much quieter." Data was collected by personal communication with dealers or from the literature and is shown in Table 10 and plotted in Figure 45.

The results tend to show that customers are not willing to pay for noise reduction voluntarily. It is also noted in the figure that people are probably not willing to pay for any noise reduction less than 3 dBA since the ear can not

Table 10. Cost of Noise Control

Reference	Company	Product	Standard dBA	Model $ Cost	Quiet dBA	Model $ Cost	Noise Reduction dBA	Cost Increase	Popular?
*	Whirlpool	Air Conditioner					much quieter	15%	Yes
*	Dayton	Ceiling Fan		20	4 sones	23		15%	Yes
*	FMC	Garden Tractor	75.5	1885	69	2247	6.5	19%	No
*	Homelite	Chain Saw	104		83		21	0	Yes
*	Chicago Pneumatic	Jack Hammer	109	695	99	800	10	15%	No
51	GM	Garbage Truck	88	13000	77.5	13102	10.5	1%	Yes
52		Typewriter						60%	
52		Dishwasher						$5/dB	
52		Chain Saw					10	2%	
52		Pneumatic Hammer					7	10%	
8		Lawn Mower						$3/dB	
17		Construction Equipment						15%/4 dB	

*Personal communication with dealer.

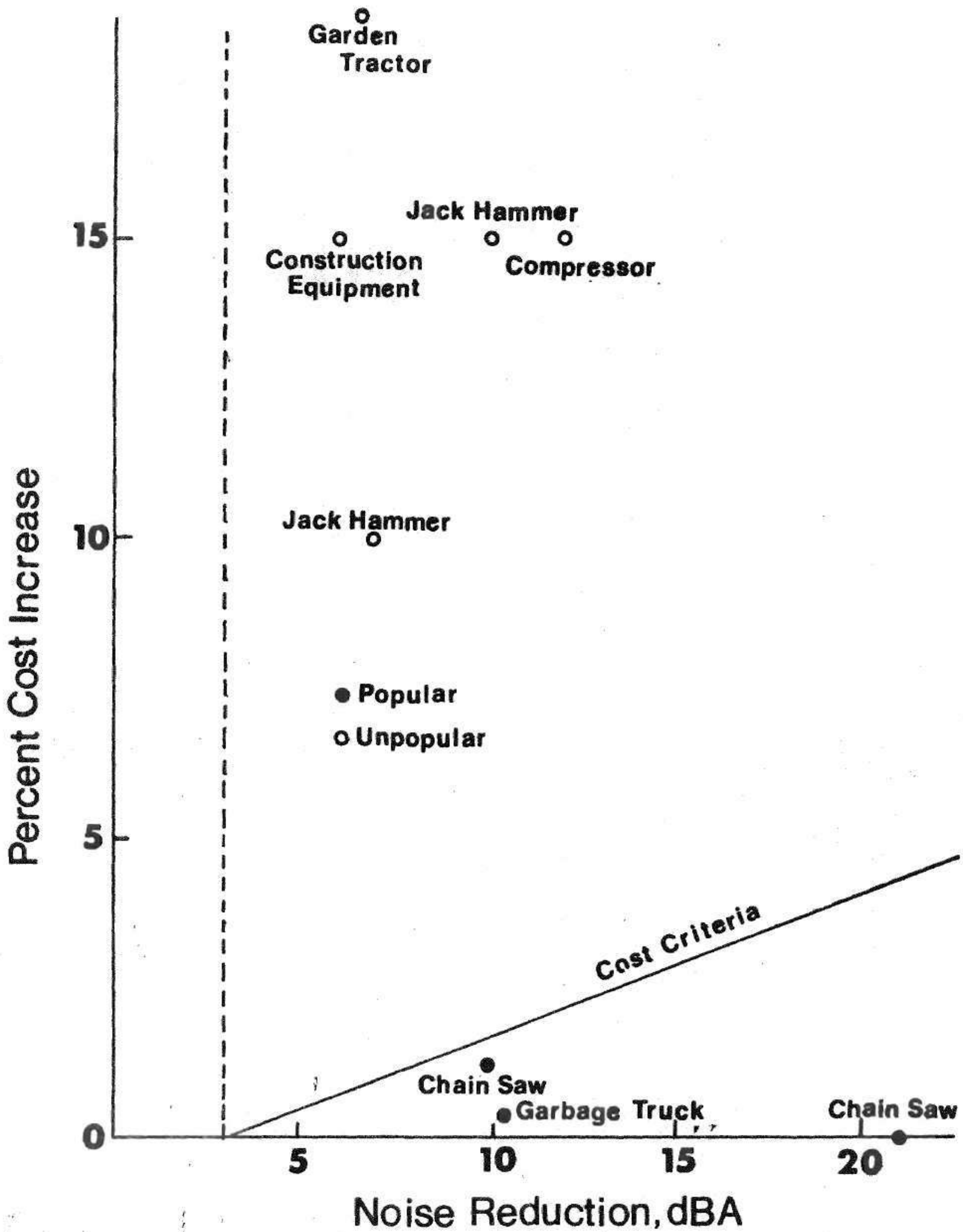


Figure 45. Cost of Noise Control

discern that small a change in noise levels. The data on the ceiling fan and the air conditioner tends to imply that up to 15 percent increase in costs will be tolerated for a decrease resulting in no annoyance whatsoever. From Figure 45 the most people are willing to pay is about .3 percent increase in cost per dBA reduction in noise levels.

APPENDIX B

DETERMINING RPM VARIATION

To determine the variation in mower rpm with time, the mower was warmed up for five minutes and the following random rpm readings were taken during two consecutive two and one half minute periods.

Table 11. RPM Data

Period (sec)	RPM	Period (sec)	RPM
.019921	3011.8	.019803	3029.9
.020183	2972.8	.019838	3024.5
.019875	3018.8	.019864	3020.5
.020106	2984.1	.019766	3035.5
.019853	3022.2	.019949	3007.7
.020101	2984.9	.019736	3040.1
.019993	3001.0	.019858	3021.5
.019927	3011.0	.020088	2986.8
.018584	3228.6	.019963	3005.6
.019846	3023.3	.019848	3023.0
.018544	3235.5	.020612	2998.2
.019753	3037.5	.018231	3291.1
.020041	2993.9	.020023	2996.6
.018433	3255.5	.019660	3051.9
.019766	3035.5	.019728	3041.4
	Σ = 45816.4		Σ = 45574.3

The average rpm of the two periods is 3054.4 and

3038.28 indicating a change of 16.1 rpm in two and one half minutes. The average rpm for the five minute test was 3046.3 rpm, hence the percent change in rpm is

$$\frac{16.1}{3046.3} \times 100 = .5\%$$

The standard deviation in rpm for the five minute test can be found from the relation below

$$S = \sqrt{\frac{N\Sigma rpm^2 - (\Sigma rpm)^2}{N^2}}$$

as 83.527 rpm or 2.74 percent of 3046.3.

APPENDIX C

DATA REPEATABILITY

The repeatability of the data was found by comparing three different data sets taken on three different occasions. The data are shown in Figure 46.

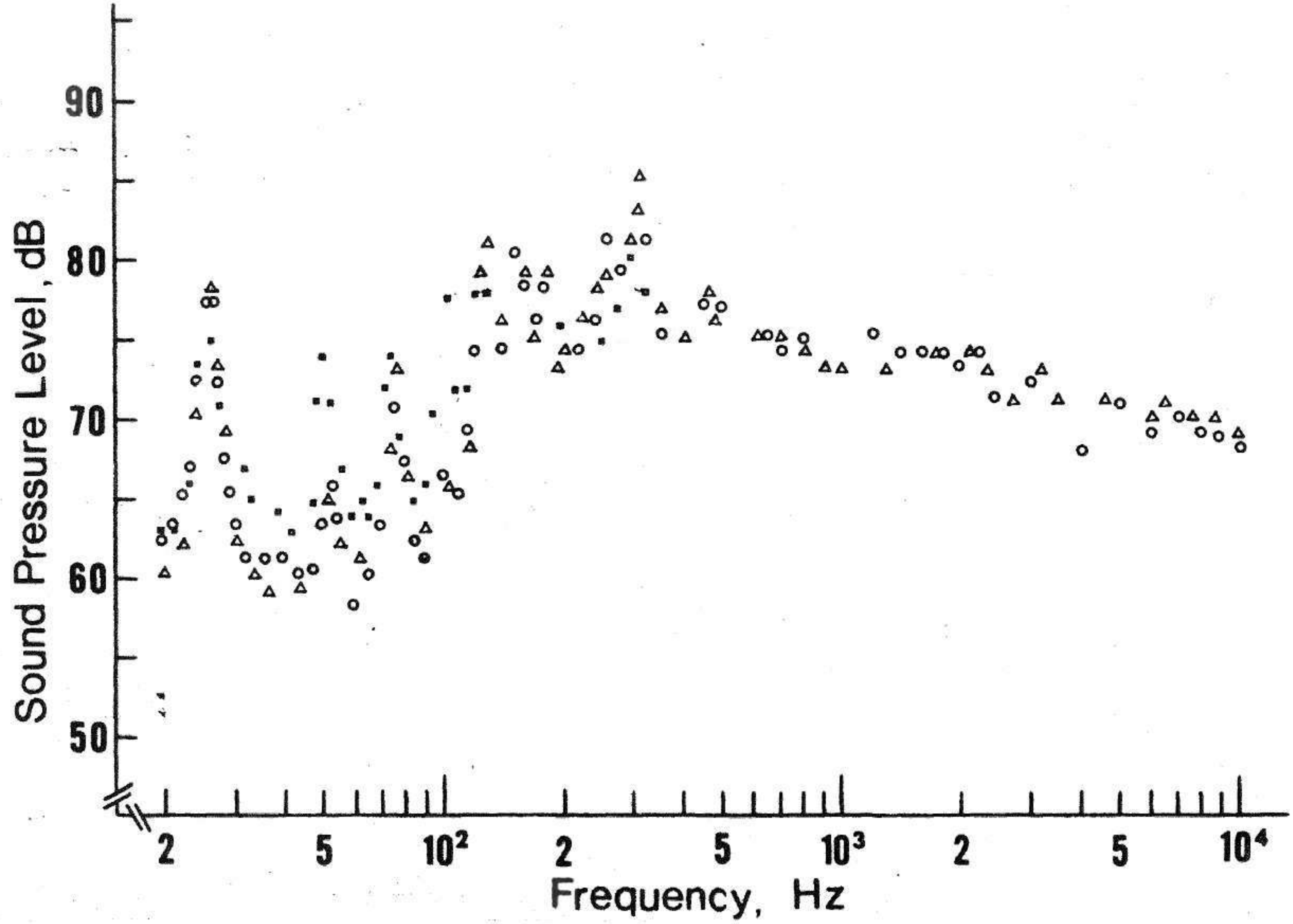


Figure 46. Data Repeatability

APPENDIX D

CALCULATING BLADE NOISE FREQUENCIES

<u>Rotation Noise</u>

Using equation (1) from Chapter IV with B equal to 2 and Ω equal to 3000 rpm or 50 revolutions per second

$$f = nB\Omega = n(2)(50) = 100n \text{ Hz} \tag{1}$$

<u>Vortex Noise</u>

Using equation (2) from Chapter IV

$$f = \frac{kv}{d} \tag{2}$$

The blade profile width, d, is .125 inches. The blade tip velocity, v, is

$$v = wl = \left(\frac{3000\text{rev}}{\text{min}} \quad \frac{2\pi}{60}\right)(9.5 \text{ in}) = \frac{2950 \text{ in}}{\text{sec}}$$

At Reynolds Numbers above 1000, such as is the case here, the Strouhal number, k, is equal to .21. Using equation (2)

$$f = \frac{kv}{d} = \frac{(.21)(2950)}{.125} = 5000 \text{ Hz}$$

Blade Bending Modes

Cantilever vibration modes may be predicted from equation (3) of Chapter IV.

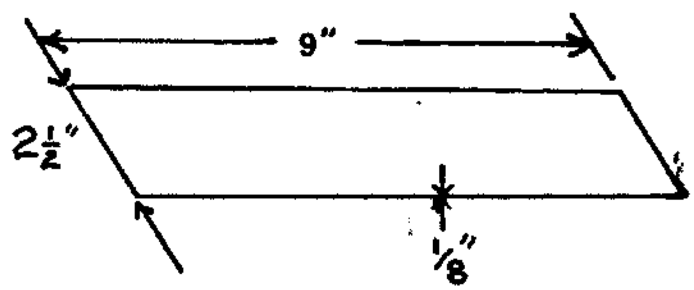
$$w_n = a_n\sqrt{\frac{EIg}{\eta l^3}} \tag{3}$$

where $a_1 = 3.52$

$a_2 = 22.0$

$a_3 = 61.7$

η = weight



$$\eta = \frac{.283\,lb}{in^3}(2.5 \times .125 \times 9) = .7956\ lb_f$$

$$I = \frac{\text{width} \times d^3}{12} = \frac{(2.5)(.125)^3}{12} = 4.07 \times 10^{-4} in^4$$

$$w_n = a_n \quad \frac{(30\times10^6)(4.07\times10^{-4})(386)}{(.7956)(9)^3}$$

$$= 90.14\ a_n$$

$$w_1 = 90.14 \times 3.52 = 317\frac{\text{rad}}{\text{sec}}$$

$$f_1 = \frac{w_1}{2\pi} = 50.49 \text{ Hz}$$

f_1 was experimentally determined to be 47.6 Hz which agrees quite well with the 50.49 Hz predicted theoretically. Since the experimentally determined frequency was considered more accurate, the radical term in equation (3) was corrected from 90.14 to 85.0. Using this constant, the natural frequencies of the higher modes were found to be as follows:
f_1 = 47.6 Hz; f_2 = 297 Hz; f_3 = 823 Hz.

APPENDIX E

VIBRATION MODEL OF MOWER

The purpose here is to find the transmissibility of the system shown in Figure 47, that is, what displacement x_1 will result from a force input to m_2.

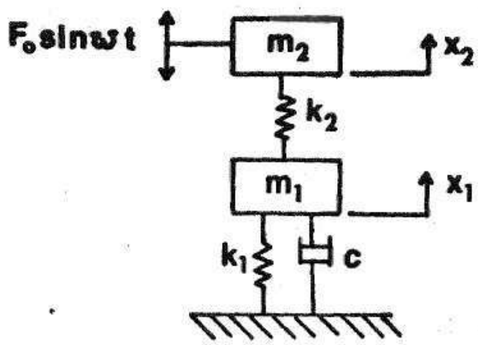


Figure 47. Mower Vibration Model

The equations of motion are as shown below.

$$
\begin{aligned}
m_1x_1 + c\dot{x}_1 + (k_2+k_1)x_1 - k_2x_2 &= 0 \\
m_2x_2 - k_2(x_1) + k_2x_2 &= F_o\sin wt
\end{aligned}
\qquad (1)
$$

Assuming a solution of the form

$$
\begin{aligned}
x &= Xe^{jwt} \\
\dot{x} &= jwXe^{jwt} \\
\ddot{x} &= -w^2Xe^{jwt}
\end{aligned}
\qquad (2)
$$

Substituting (2) into (1) yields

$$(-w^2 m_1 + (k_1+k_2) + jwc) X_1 - k_2 X_2 = 0$$

$$(-w^2 m_2 - k_2) X_1 + k_2 X_2 = F_o$$

X_1 may be found by using Cramer's Rule. The solution of this set of equations may be found in any elementary vibrations text, for example see reference 28. Knowing X_1, the transmissibility, $\frac{X_1}{F_o/k_1}$ may be found as

$$\frac{X_1}{F_o/k_1} = f(m_1 m_2, \frac{k_1}{m_1}, \frac{k_2}{m_2}, \frac{c}{c_c}, w)$$

A simplification results if it is assumed that the motion of shroud is much less than that of the engine when the isolator is in place. The motion assumption may be justified by considering the respective accelerations in Figure 22. Also implicit in this assumption is the fact

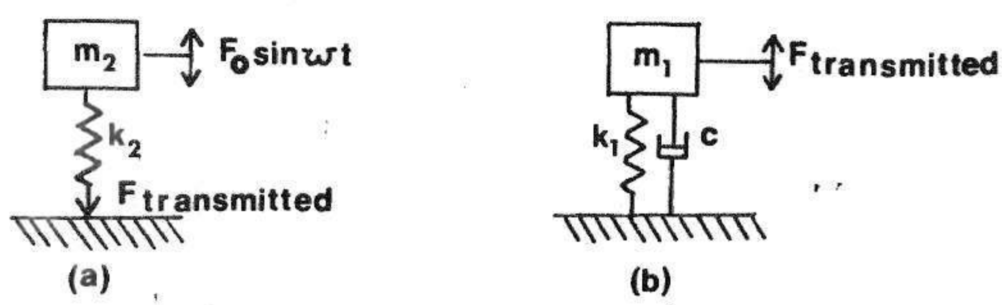


Figure 48. Simplified Mower Vibration Model

that the mass of the engine is much less than that of the shroud. The problem is now decoupled and is presented below as shown in reference 30.

Once $F_{transmitted}$ is found for the system (a) of Figure 48, it may be applied to system (b) to find its displacement. The transmissibility will then be found as

$$\frac{x_1}{F_o/k_1} = f\left(\frac{k_1}{m_1}, \frac{k_2}{m_2}, \frac{c}{c_c}, w\right)$$

The first problem has the well known solution

$$F_{transmitted} = F_o\sqrt{\frac{1}{\left[1 - \frac{w^2}{\Omega x_2}\right]}} \qquad (3)$$

where

$$\Omega x_2 = \frac{k_2}{m_2}$$

The second problem has the solution

$$x_1 = \frac{F_T k_1}{\sqrt{[1-w^2/\Omega x_1]^2+[2(w/\Omega x_1)(C/Cc)]^2}}, \qquad (4)$$

where

$$\Omega_s = \frac{k_1}{m_1}$$

$$c_{cs} = 2\sqrt{k_1 m_1}$$

Substituting (3) into (4)

$$\frac{x_1}{F_o/k_1} = \frac{1}{\sqrt{[(1-\frac{w^2}{\Omega x_2})^2[(1-\frac{w^2}{\Omega x_1}) + (2\frac{c}{c_{cs}}\ \frac{w}{\Omega x_1}]}} \tag{5}$$

Equation (5) is plotted in Figure 21 for $\frac{c_c}{c_{cs}}$ = .01 and various values of $\frac{\Omega x_2}{\Omega x_1}$.

APPENDIX F

CALIBRATING MICROPHONE PROBE

The Bruel and Kjaer Type UA 0040 microphone probe kit is a device capable of measuring sound pressure levels in a hostile environment such as in an exhaust pipe or in small places such as in the ear. The device consists of a tube and adapters to connect it to a Bruel and Kjaer 1/2 inch or 1/4 inch condensor microphone. Also included in the kit is a chamber with which the probe may be calibrated. The calibration setup is shown in Figure 49.

The sound source in the calibrator chamber is an earphone. The signal used to drive the source was a Bruel and Kjaer Type 1042 Sine Wave Generator. The sound pressure level in the chamber was monitored with a Bruel and Kjaer Type 4134 1/2 inch condensor microphone which was powered by a Bruel and Kjaer Type 2801 Microphone Power Supply. The

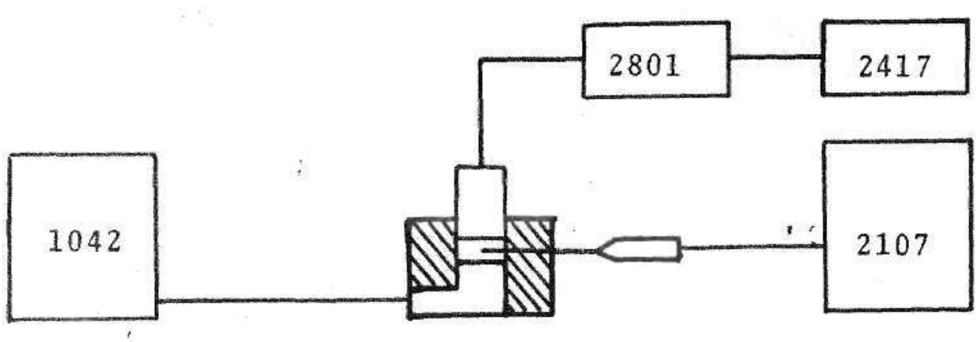


Figure 49. Microphone Probe Calibration Setup

microphone output was monitored with a Bruel and Kjaer Type 2417 Random Noise Voltmeter. The probe microphone output was monitored with the Bruel and Kjaer Type 2107 Wave Analyzer.

The calibration procedure is given in the following. The SPL in the chamber was set by adjusting the 1042 output until the 2417 showed 120 dB in the chamber. The output of the probe microphone was then recorded. When the excitation frequency was changed, the SPL in the chamber was again adjusted to 120 dB and the output of the probe at the new frequency was recorded.

All of the probes tested were 100 mm long. Three different diameter probes were tried, 2 mm, 1 mm, and .5 mm. The .5 mm dia probe had the most linear response of the three probes at frequencies between 10 and 3000 Hz. The probe calibration curve is shown in Figure 50. The calibration was found to be the same for levels between 100 and 135 dB SPL. The earphone was not capable of driving the chamber above 135 dB SPL.

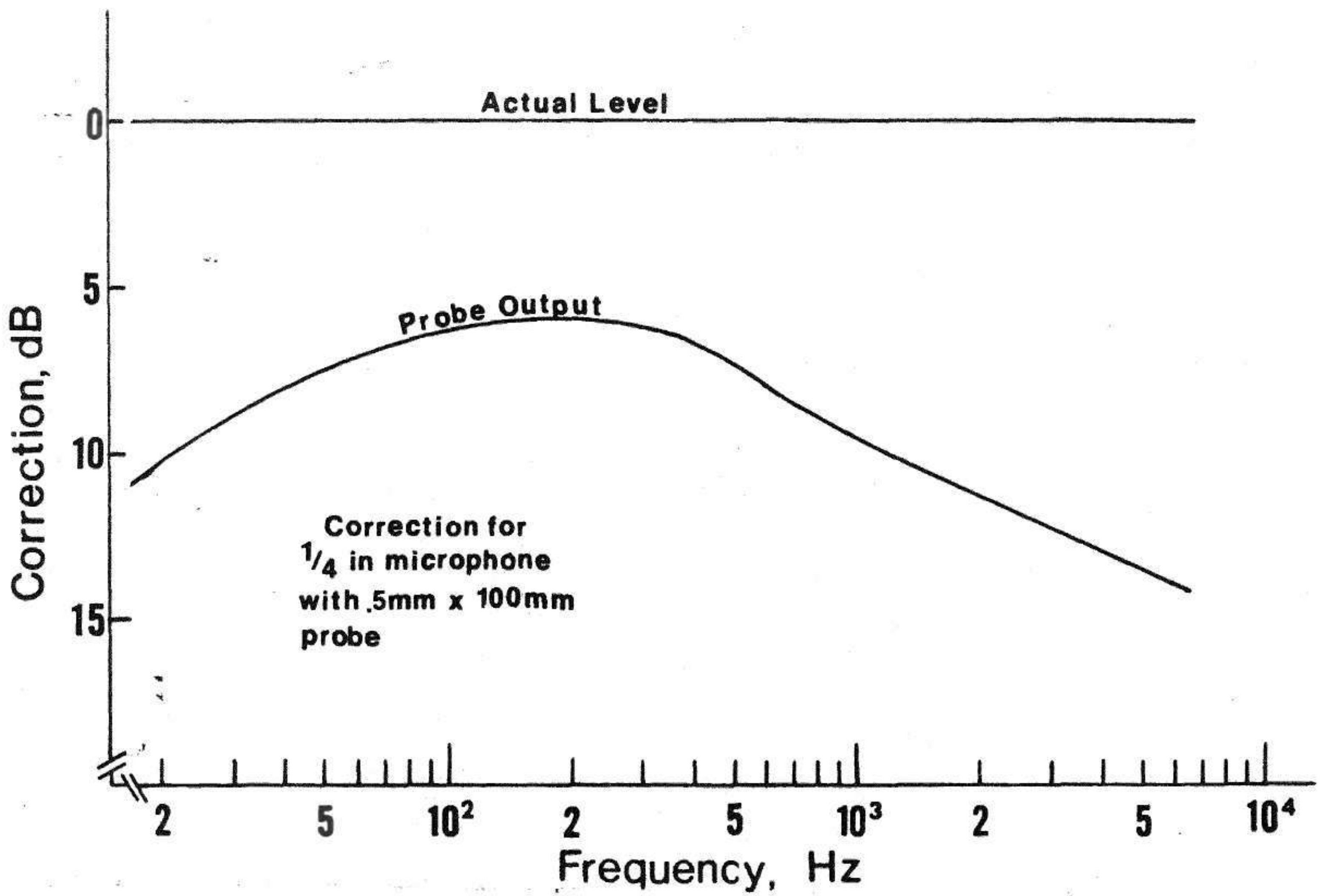


Figure 50. Microphone Calibration Curve

APPENDIX G

ATTENUATION EQUATIONS

Equation (1) shown below is the classic wave equation

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} \qquad (1)$$

which describes the propagation of plane waves in a duct. It has the general solution

$$p = p_+ c^{jw(t-\frac{x}{c})} + p_- e^{jw(t+\frac{x}{c})} \qquad (2)$$

where p_+ and p_- denote waves traveling backward and forward, respectively. Let

$$A = p_+ e^{jwt}$$

$$B = p_- e^{jwt}$$

$$k = \frac{w}{c} = \frac{2\pi f}{c}$$

then (2) becomes

$$p = Ae^{-jkx} + Be^{jkx} \qquad (3)$$

Consider an acoustic wave with amplitude A_1 and impedance Z_{01} traveling along a duct of cross sectional area S_1 as shown in Figure 51. Define the origin of coordinates to be at I. At I, (x = 0), the area of the duct abruptly changes from S_1 to S_2 and a wave with amplitude B_1 is reflected backward and a wave of amplitude A_2 travels on into the expansion chamber. According to (3), the pressure at x=0 is $p = A_1 + B_1$. Since the duct has impedance $Z_{01} = \frac{\rho_o c}{S_1} = \frac{p}{u}$ and the volume velocity is $U_r = \frac{-B}{Z_{01}}$ the volume current at x=0 will be

$$u = \frac{A_1 - B_1}{Z_{01}}$$

Assume for the moment that both ends of the duct are infinitely long, i.e., there is no reflection at II. The amplitude of the reflected wave may be found by assuming continuity of pressure and volume current at junction I. Continuity of pressure at x=0 yields

$$A_1 + B_1 = A_2 \tag{4}$$

Continuity of volume current yields

$$\frac{A_1 - B_1}{Z_{01}} = \frac{A_2}{Z_{02}} \tag{5}$$

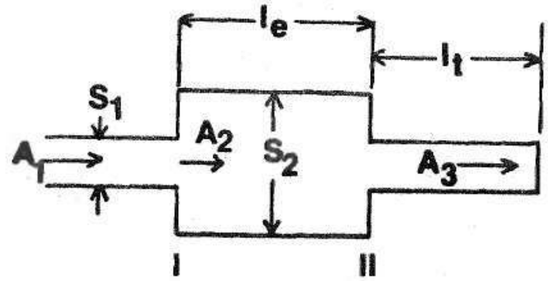


Figure 51. Schematic of Single Expansion Chamber

where

$$Z_{02} = \frac{\rho_o c}{S_2}$$

Letting $\frac{S_2}{S_1} = m$ and combining (8) and (9) yields

$$\frac{A_1}{A_2} = \frac{2}{m+1} \qquad (6)$$

The transmission factor α_t is defined to be the square of the ratio of the pressure incident on the filter to the pressure transmitted. According to equation (1), Chapter VI, the attenuation of the filter is

$$\text{Attenuation} = 10 \log_{10}\left(\frac{1}{\alpha_t}\right) \qquad (7)$$

Note from (6) that as the area ratio increases, the attenuation increases. Experimental results [47] verify that the attenuation of an expansion chamber is a function of the area ratios.

Consider the case where there is second area change back to the area S_1 at a distance l_e from $x = 0$ which causes a reflected wave B_2 that moves back toward I. Continuity of pressure and volume current at I yields

$$A_1 + B_1 = A_2 + B_2 \tag{8}$$

$$A_1 + B_1 = m(A_2 - B_2) \tag{9}$$

Setting the pressure at junction II equal to A_3, the amplitude of a wave which proceeds out the tailpipe without reflection yields

$$A_2 e^{-jkle} + B_2 e^{jkle} = A_3 \tag{10}$$

Continuity of volume current at II yields

$$m(A_2 e^{-jkle} - B_2 e^{jkle}) = A_3 \tag{11}$$

Combining (10)-(14) results in

$$\text{Attenuation} = 10 \log_{10}[1+\tfrac{1}{4}(m-\tfrac{1}{m})^2 \sin^2 kle] \tag{12}$$

Note that the attenuation is a function of the area ratio and the factor kl_e.

Using a similar approach, the attenuation of multiple expansion chamber systems can also be found. With a system having two chambers of equal length, the attenuation is a function of kl_e, m and the length of the connector between the two chambers.

Another system that must be considered is the side branch resonator shown in Figure 52. Choosing the origin of coordinates at the branch, continuity of pressure and volume current yields

$$p_i + p_r = p_b = p_t \tag{13}$$

$$u_b = \frac{p_b}{Z_b} \quad u_i = \frac{p_i}{\frac{\rho_o c}{S_1}}; \; u_r = \frac{p_r}{\frac{\rho_o c}{S_2}}; \; u_t = \frac{p_t}{\frac{\rho_o c}{S_2}}$$

$$\frac{1}{Z_{01}}(p_i - p_r) = p + \left(\frac{1}{Z_b} + \frac{1}{Z_o}\right) \tag{14}$$

Solving (13) and (14) for the ratio pi/pr results in

$$\text{Attenuation} = 10 \log_{10}\left[\frac{\left(R_b + \frac{Z_o}{2}\right)^2 + X_b^2}{R_b^2 + X_b^2}\right] \tag{15}$$

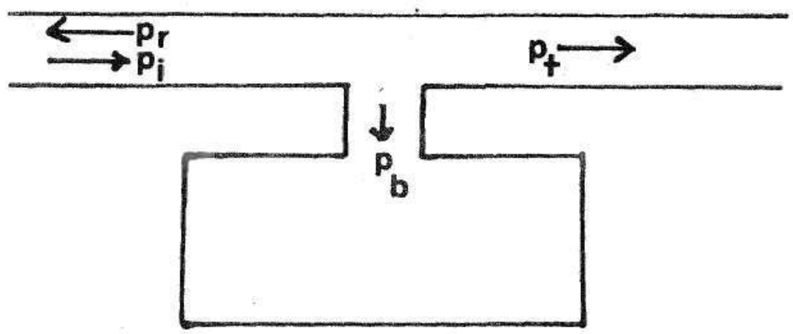


Figure 52. Schematic of Side Branch

where R_b and X_b refer to the real and imaginary components of the branch impedance. X_b is given by (4) in Chapter VI as

$$X_b = wM - \frac{1}{wC}$$

Using values of M and C from (2) and (3) in Chapter VI

$$X_b = \frac{w\ 1'}{\pi a^2} - \frac{\rho c^2}{wV}$$

Replacing $\frac{a^2}{1'}$ by C_o, the conductivity,

$$X_b = \frac{w\rho}{C_o} - \frac{\rho c^2}{wV} \tag{16}$$

Define $f = \frac{\omega}{2\pi}$ at $X_b = 0$ to be f_r. When $X_b = 0$

$$f_r = \frac{c}{2\pi}\sqrt{\frac{C_o}{V}} \tag{17}$$

If the viscosity in the branch is neglected, (15) simplifies to

$$\text{Attenuation} = 10 \log_{10}[1+\frac{Z_0^{\ 2}}{4X_b^{\ 2}}] \qquad (18)$$

Combining (16)-(18) after some algebra yields

$$\text{Attenuation} = 10 \log_{10}[1+(\frac{\sqrt{C_o V}}{2S}/(\frac{f}{f_r}-\frac{f_r}{f}))^2] \qquad (19)$$

Note that the attenuation is a function of the two parameters

$$\frac{\sqrt{C_o V}}{2S} \text{ and } \frac{f}{f_r}$$

The previous discussion has neglected the effects of the tailpipe. Levine and Schwinger [45] have considered the radiation of sound from a circular pipe and show that there is a reflection coefficient of nearly -1 for low frequencies, that is, any wave incident on the pipe is reflected 180° out of phase. The assumption made therefore is that all of the sound in the tailpipe completely reflects with negative sign, that is, a compression wave is reflected as a rarefaction.

Only one system with a finite tailpipe will be considered here, however, the analysis for the other system is exactly the same. Consider the system shown in Figure with a tailpipe of length l_t. Continuity of pressure and volume current at I yields

$$A_1 + B_1 = A_2 + B_2 \tag{20}$$

$$A_1 - B_1 = m(A_2 - B_2) \tag{21}$$

At II, assuming total reflection from the tailpipe, continuity of pressure and volume current yields

$$A_2 e^{-jkl_e} + B_2 e^{jkl_e} = A_3 - A_{3e}{}^{-i2kl_t} \tag{22}$$

$$m(A_2 e^{-ikl_e} - B_2 e^{ikl_e}) = A_3 + A_3 e^{-i2kl_t} \tag{23}$$

Solving (20)-(23) for $\frac{A_1}{A_3}$, the attenuation is found to be

$$\text{Attenuation} = 10 \log_{10} \left|\frac{A_1}{A_3}\right|^2$$

where

$$\frac{A_1}{A_3} = 1 + \frac{(m^2-1)^2}{2m^2}\sin^2 kl_e - \frac{m^2-1}{2m}\sin^2 kl_t \sin^2 kl_e -$$

$$\frac{m^4-1}{2m^2}\cos 2kl_t \sin^2 kl_e \tag{24}$$

Solving (24) for the cutoff frequency, that is, the frequency below which no attenuation will occur

$$f_c = \frac{c}{2\pi}\sqrt{\frac{4+\frac{2l_e}{ml_t}}{(m+\frac{1}{m})l_e l_t}} \tag{25}$$

APPENDIX H

COMPUTER PROGRAM FOR MUFFLER DESIGN

The computer program shown on the next page was used as a tool for the design of single chamber concentric resonator systems.

```
NEW:RESONATOR
 READY

**TAPE START**

90  D3=100
100 L=100
110 V1=100
120 C=1880
130 K=40
140 S=.00305
150 FOR L1=.3 TO .8 STEP .01
160 FOR F1=40 TO 100 STEP 10
170 K1=2*3.14159*F1*L1/C
180 FOR V=.1 TO .4 STEP .01
190 C1=39.4784*F1**2*V/C**2
200 A1=(C1*V)**.5/(2*S)
210 F2=F1/(1+C1*L1/(2*S))**.5
220 IF F2>K GO TO 430
230 FOR F=100 TO 300 STEP 10
240 M=F/F1-F1/F
250 IF ABS(M)<.01 GO TO 350
260 IF ABS(2*K1*F/F1)<.001 GO TO 350
270 A=2*A1*SIN(2*K1*F/F1)/M
280 A=A+4*A1**2*SIN(K1*F/F1)**2/M**2
290 A=A+1
300 IF A>0 GO TO 330
310 A2=0
320 GO TO 340
330 A2=10*LGT(A)
340 IF A2<10 GO TO 430
350 NEXT F
355 D=(V/(.785*L1)+S)**.5
360 IF V<V1 GO TO 380
370 GO TO 430
380 L=L1
390 F3=F1
400 C3=C1
410 V1=V
420 D3=D
430 NEXT V
440 NEXT F1
450 NEXT L1
460 PRINT'OPTIMUM SINGLE CHAMBER RESONATOR PARAMETERS'
470 PRINT'CUTOFF FREQUENCY=';K;'HZ'
480 PRINT'FR','CO','VOLUME','DIAMETER','LENGTH'
490 PRINT'HZ','FEET','CUBIC FT','FEET','FEET'
500 PRINT
510 PRINT F3,C3,V1,D3,L
520 END

END OF TAPE
 READY

RUN


R              16:22:14    2 MAR 73


OPTIMUM SINGLE CHAMBER RESONATOR PARAMETERS
CUTOFF FRECUENCY= 40 HZ
FR              CO              VOLUME          DIAMETER        LENGTH
HZ              FEET            CUBIC FT        FEET            FEET

 60              .010053         .25             .649686         .76

 TIME :  13.065
```

Figure 53. Computer Program

APPENDIX I

EFFECT OF MULTIPLE HOLES IN PIPE MUFFLER

Figure 54 shows the negative attenuation resulting from the addition of holes in the pipe muffler for both coherent and incoherent sound. Figure 55 shows the attenuation per hole at 200 Hz as a function of hole radius. The resultant attenuation will be

$$\text{Attenuation} = \text{Attenuation per hole} - \text{correction for \# of holes} \qquad (1)$$

For a total hole area of .333 in^2, the number of holes and their radius are related by

$$\text{\# of holes} = \frac{.333}{\pi r^2} = \frac{.106}{r^2} \qquad (2)$$

Combining (1) and (2) and Figures 54 and 55 it can be easily shown that for the incoherent case

$$\text{Attenuation} = 28.25 + 7.5 \log(r), \qquad (3)$$

and for the coherent case

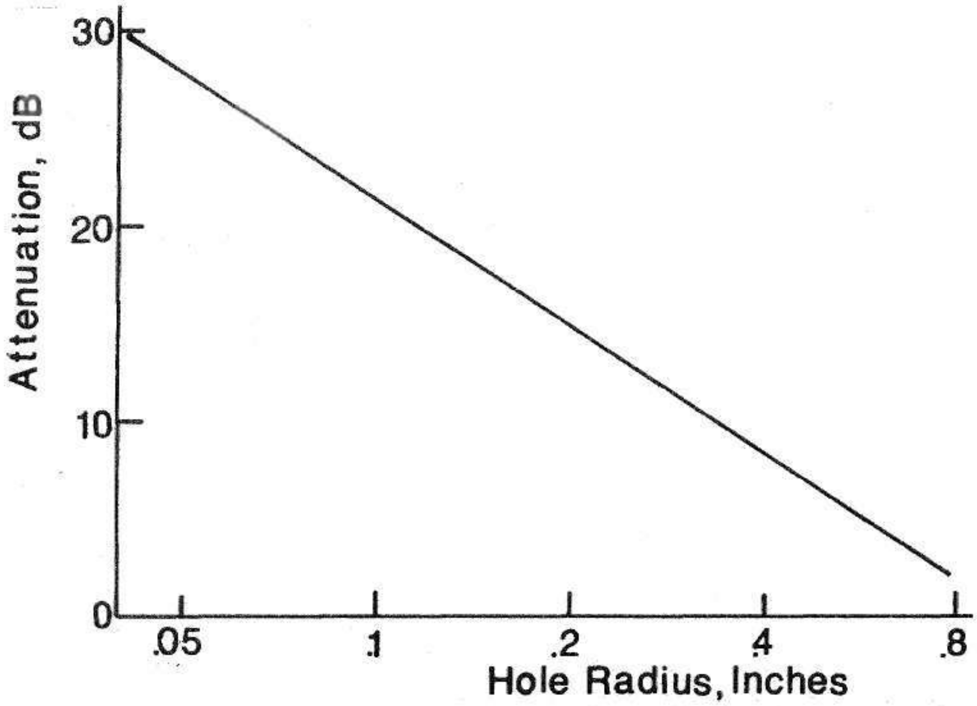


Figure 54. Decrease in Attenuation with Increase in Number of Holes in Pipo Muffler

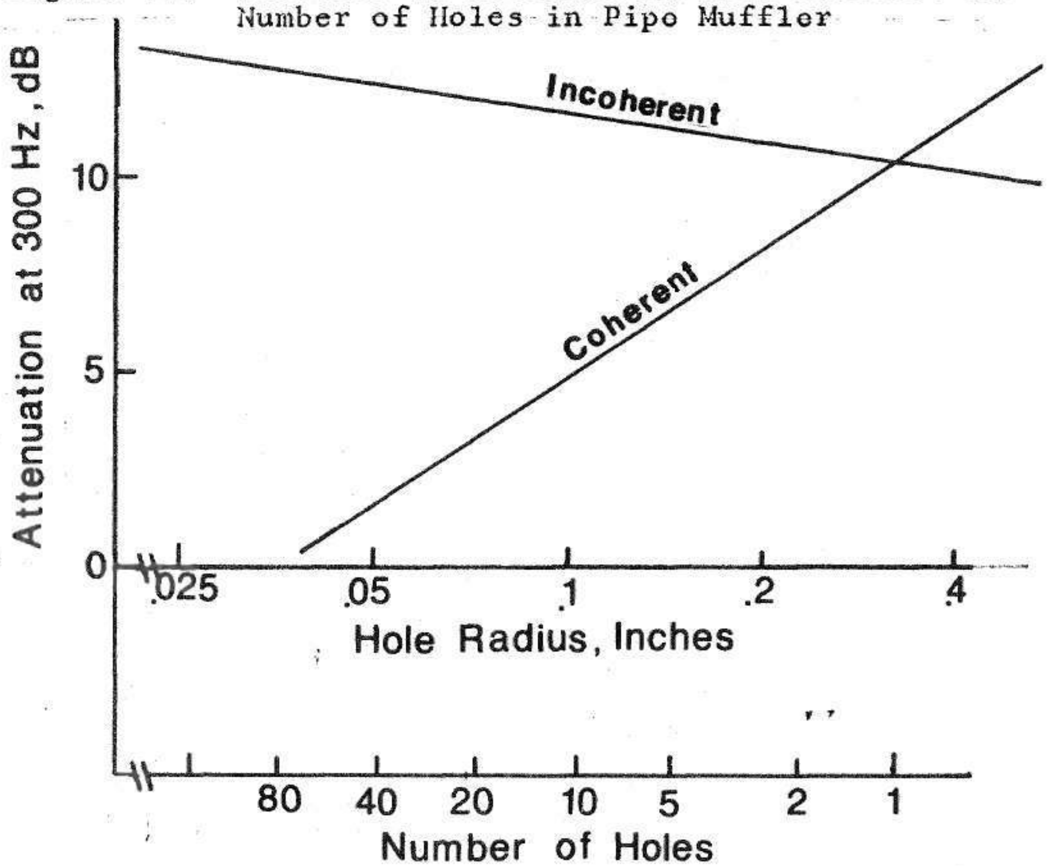


Figure 55. Attenuation vs. Hole Size

$$\text{Attenuation} = 9.25 - \log(r) \tag{4}$$

Equations (3) and (4) are plotted in Figure 56. Note that for the case of incoherent sound the attenuation is found to increase as the number of holes increases and their size decreases. For the case of coherent sound the attenuation decreases as the number of holes increases.

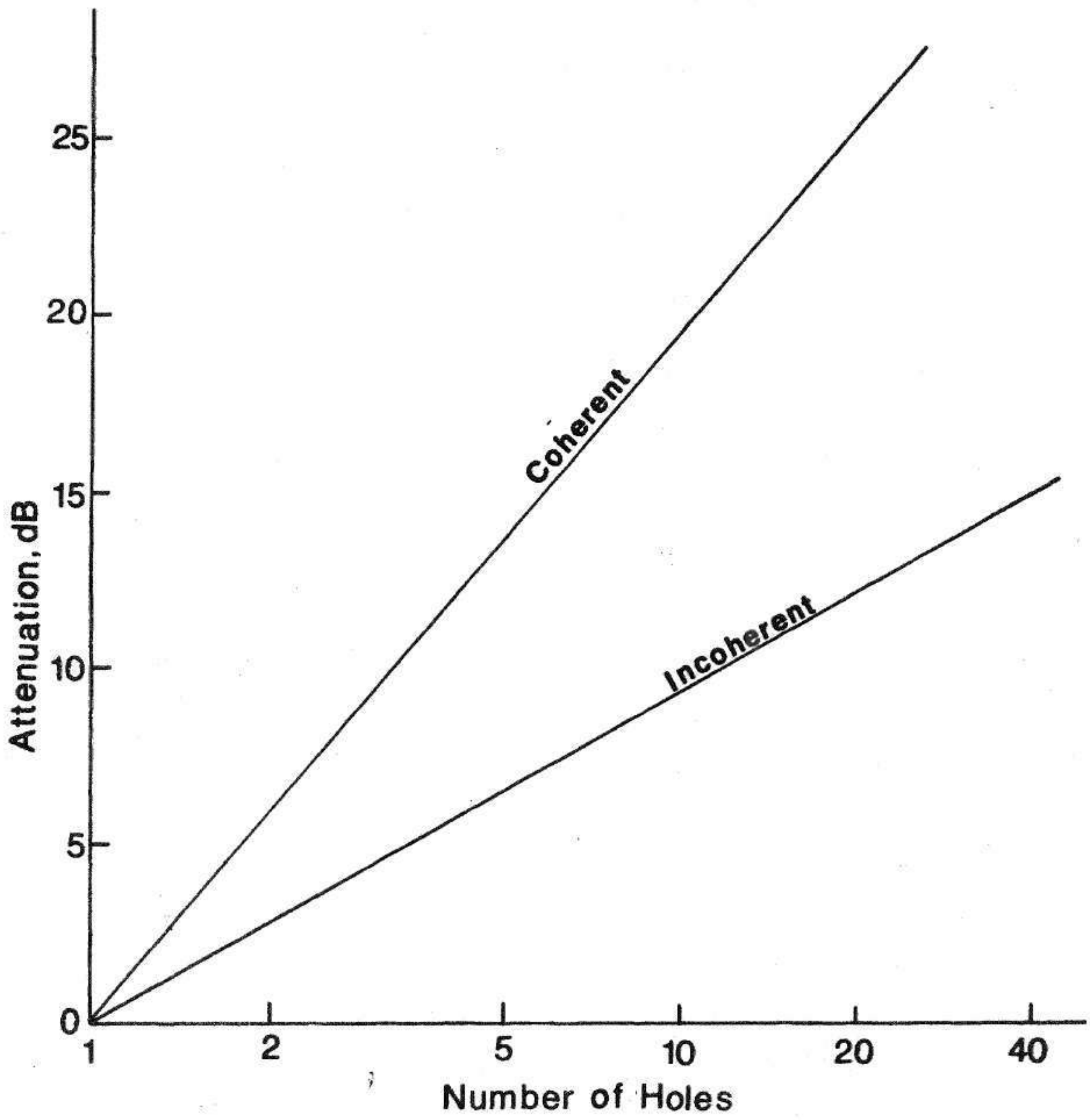


Figure 56. Attenuation as a Function of Hole Size for Pipe Muffler

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